Why and When to Teach Definitions: Necessary and Unnecessary Discontinuities Resulting from the Definition of Mathematical Concepts

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Abstract : We examine reasons for introducing definitions in teaching mathematics in a number of different cases. We try to determine if, where, and when to provide a definition, and which definition to choose. We characterize different types of definitions and the different purposes we may have for formulating them, and detail examples of each type. Giving a definition at a certain stage can sometimes be detrimental to the development of the concept image. In such a case, it is advisable to delay the precise definition to a later stage. We describe two models, the 'successive approximation model', and the 'model of the extending definition' that fit such situations. Detailed examples that fit the different models are given based on material taken from a number of textbooks, and analysis of the way the concept is introduced, and where and how its definition is given. Our conclusions, based on this analysis, is that some of the definitions given may cause discontinuities in the learning sequence and constitute obstacles and unnecessary cognitive conflicts in the formation of the concept definition. However, in other cases, the discontinuity in passing from definition to definition actually serves a didactic purpose, is unavoidable for the mathematical evolution of the concept image, and is essential for students to deepen their understanding.

Keywords : concept image, mathematical definitions, mathematics education, mathematics teaching

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