Dependence of the Photoelectric Exponent on the Source Spectrum of the CT

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Abstract : X-ray attenuation coefficient [$\mu(E)$] of any substance, for energy (E), is a sum of the contributions from the Compton scattering [$\mu_{Com}(E)$] and photoelectric effect [$\mu_{Ph}(E)$]. In terms of the, electron density (ρ_e) and the effective atomic number (Z_{eff}) we have $\mu_{Com}(E)$ is proportional to [$(\rho_e)f_{KN}(E)$] while $\mu_{Ph}(E)$ is proportional to [$(\rho_e Z_{eff}^x)/E^y$] with $f_{KN}(E)$ being the Klein-Nishina formula, with x and y being the exponents for photoelectric effect. By taking the sample's HU at two different excitation voltages ($V=V1, V2$) of the CT machine, we can solve for $X=\rho_e$, $Y=\rho_e Z_{eff}^x$ from these two independent equations, as is attempted in DECT inversion. Since $\mu_{Com}(E)$ and $\mu_{Ph}(E)$ are both energy dependent, the coefficients of inversion are also dependent on (a) the source spectrum $S(E,V)$ and (b) the detector efficiency $D(E)$ of the CT machine. In the present paper we tabulate these coefficients of inversion for different practical manifestations of $S(E,V)$ and $D(E)$. The HU(V) values from the CT follow: $\langle\mu(V)\rangle=\langle\mu_w(V)\rangle[1+HU(V)/1000]$ where the subscript 'w' refers to water and the averaging process $\langle....\rangle$ accounts for the source spectrum $S(E,V)$ and the detector efficiency $D(E)$. Linearity of $\mu(E)$ with respect to X and Y implies that (a) $\langle\mu(V)\rangle$ is a linear combination of X and Y and (b) for inversion, X and Y can be written as linear combinations of two independent observations $\langle\mu(V1)\rangle$, $\langle\mu(V2)\rangle$ with $V1 \neq V2$. These coefficients of inversion would naturally depend upon $S(E, V)$ and $D(E)$. We numerically investigate this dependence for some practical cases, by taking $V = 100$, 140 kVp, as are used for cardiological investigations. The $S(E,V)$ are generated by using the Boone-Seibert source spectrum, being superposed on aluminium filters of different thickness l_{Al} with $7mm \leq l_{Al} \leq 12mm$ and the $D(E)$ is considered to be that of a typical Si[Li] solid state and GdOS scintilator detector. In the values of X and Y, found by using the calculated inversion coefficients, errors are below 2% for data with solutions of glycerol, sucrose and glucose. For low Z_{eff} materials like propionic acid, Z_{eff}^x is overestimated by 20% with X being within 1%. For high Z_{eff}^x materials like KOH the value of Z_{eff}^x is underestimated by 22% while the error in X is + 15%. These imply that the source may have additional filtering than the aluminium filter specified by the manufacturer. Also it is found that the difference in the values of the inversion coefficients for the two types of detectors is negligible. The type of the detector does not affect on the DECT inversion algorithm to find the unknown chemical characteristic of the scanned materials. The effect of the source should be considered as an important factor to calculate the coefficients of inversion.