

Ion-Acoustic Double Layer in a Plasma with Two-Temperature Nonisothermal Electrons and Charged Dust Grains

Basudev Ghosh, Sreyasi Banerjee

Abstract—Using the pseudopotential technique the Sagdeev potential equation has been derived in a plasma consisting of two-temperature nonisothermal electrons, negatively charged dust grains and warm positive ions. The study shows that the presence of nonisothermal two-temperature electrons and charged dust grains have significant effects on the excitation and structure of the ion-acoustic double layers in the model plasma under consideration. Only compressive type double layer is obtained in the present plasma model. The double layer solution has also been obtained by including higher order nonlinearity and nonisothermality, which is shown to modify the amplitude and deform the shape of the double layer.

Keywords—Two temperature non-isothermal electrons and charged dust grains.

I. INTRODUCTION

A Double Layer (DL) in plasma consists of two oppositely charged parallel layers resulting in a strong electric field across the layer which can accelerate the plasma electrons and ions in opposite directions producing an electric current. DLs occur naturally in a variety of space plasma environments [1]–[4] and are of considerable interest in astrophysics [5]–[9]. In last few years, solitary waves and double layers have been studied by various authors considering both dust acoustic and ion acoustic waves in magnetized and unmagnetized plasmas [10]–[15]. Usually in the study of ion-acoustic, dust acoustic or dust ion-acoustic solitons and double layers one usually considers inertial ions and Boltzmann distributed electrons. However, in practice, the electrons may not have a Maxwellian distribution due to trapping of electrons in the ion-acoustic wave potential. These trapped electrons interact strongly with the wave during the evolution of the wave and therefore the electrons follow a trapped particle distribution [16], [17]. Nonisothermal distributions of electrons can be found in many astrophysical and space environments. Recently the effects of nonisothermal electrons and its relevant nonlinearity in complex plasma and fluid dynamics have been studied in detail by several authors [18]–[23]. Gill et al. [20] have studied the effects of nonisothermal electrons on ion-acoustic solitary waves in a plasma with positive and negative ions. They have shown that only compressive solitons are obtained with nonisothermal electrons even in

presence of negative ions. In some experimental situations such as hot turbulent plasmas in thermonuclear devices, hot cathode discharge plasmas, strong electron beam – plasma interaction experiments electrons can be found to have two-temperatures. Two electron-temperature plasmas are also observed in space [21]. Presence of two-temperature electrons in the ionospheric plasma has been known to the scientific community for many decades [24]. The presence of two-temperature electrons in plasma gives rise to many interesting characteristics in nonlinear propagation of waves including the excitation of ion-acoustic solitary waves and double layers in plasmas [23], [25]–[27]. Singh and Malik [23] have studied the effects of two-temperature nonisothermal electrons on ion-acoustic solitary waves in inhomogeneous magnetized negative ion plasma. They found that unlike the usual case of negative ion containing plasmas rarefactive solitons do not occur in their plasma model. Jones et al [21] have studied the effects of two-temperature electrons on ion-acoustic waves and shown that the presence of even a small fraction of lower temperature electrons could largely influence the behavior of ion-acoustic solitary waves. Goswami and Buti [28] found that the presence of lower temperature electrons results in larger amplitude of ion-acoustic solitary waves of a given width. The presence of two-temperature nonisothermal electrons in dusty plasma is also an important area of research. During the past years dusty plasma has become a growing field of research because of its existence in various environments like cometary tails, planetary rings, asteroids, magnetosphere, lower ionosphere, interstellar and circumstellar clouds and laboratory devices [12], [29]–[32]. Angelis et al. [33] show the presence of charged dust grains in plasma can modify the existing plasma wave spectra and introduce a number of new novel eigen modes such as dust acoustic waves by [34], dust ion-acoustic waves, dust lattice waves were introduced by [35] etc. Nonisothermal distributions of electrons can be found during the condensation of dust grains in dusty plasmas. Thus it would be interesting to study the presence of two-temperature nonisothermal electrons in dusty plasmas. Recently a few authors have considered the presence of nonisothermal electrons and ions in dusty plasmas [36]–[40]. Gill et al. [37] considered higher order nonlinear effects on ion-acoustic solitary waves in a four component dusty plasma with nonisothermal electron distribution. They have shown that higher order nonlinearity modifies the amplitude and deforms the shape of dust acoustic solitary waves. Also, it has been shown that unlike the isothermal case only compressive

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solitons result with nonisothermal distribution. Schekinov [40] studied analytically the effects of nonisothermal electrons on dust acoustic waves in a plasma containing charged dust grains. Mamun [37] considered the effects of nonisothermal ions on small amplitude dust acoustic waves. El-Labany et al. [36] have studied dust acoustic solitary waves and double layers in a dusty plasma with two-temperature trapped ions and isothermal electrons. The effect of two-temperature ions is shown to provide the possibility for the coexistence of rarefactive and compressive dust acoustic solitary structures. The effects of nonisothermal electrons on dust ion-acoustic solitary waves have been studied by [39] in pair-ion plasmas in presence of dust in the background. Their analysis shows that nonisothermal electrons induce some kind of inertia in the propagation of nonlinear dust acoustic solitary waves and make the amplitude of the soliton smaller than that for isothermal electrons. So far as we know the effects simultaneous presence of two-temperature electrons and charged dust grains on ion-acoustic double layers have not been considered by anyone. The purpose of the present work is to address this problem.

The paper is organized in the following way: In Section II, we give the basic equations with underlying assumptions. In Section III, the nonlinear equation describing the plasma dynamics is derived by pseudo potential technique. Also here the DL solutions are obtained. Numerical results are presented and discussed in Section IV.

II. BASIC EQUATION

We consider an unmagnetized, collisionless plasma consisting of warm ions, nonisothermal two-temperature electrons and cold negatively charged dust grains. The dust grains are assumed to be of uniform mass and behave like point charged particles. Moreover, we assume that the charging of the dust particles is mainly caused by the attachment of the ions and electrons to the dust grains via collisions. The effects of photo-ionization radiation etc. for charging of the dust particles are neglected. Electrons because of their lighter mass and faster motion are initially attached to the dust particles at a faster rate than the ions, as a result the dust particles get negatively charged. The normalized equations governing the plasma particle dynamics are the following:

For ions:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + 3\sigma_i n_i \frac{\partial n_i}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (2)$$

For dust particles:

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0 \quad (3)$$

$$\frac{\partial u_d}{\partial t} + u_i \frac{\partial u_d}{\partial x} = \frac{z_d}{\mu_d} \frac{\partial \phi}{\partial x} \quad (4)$$

Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2} = n_{el} + n_{eh} + z_d n_d - n_i \quad (5)$$

where n_i , n_d , n_{eh} , n_{el} are the number densities of ions, dust particles, hot electrons and cold electrons respectively; u_i and u_d are respectively the velocities of ions and dust particles; $\sigma = T_i/T_{eff}$ in which T_i and T_{eff} denote respectively the ion temperature and some effective electron temperature; the effective electron temperature is defined as [31]

$$T_{eff} = \frac{(n_{el0} + n_{eh0})T_{el}T_{eh}}{(n_{el0}T_{eh} + n_{eh0}T_{el})} \quad (6)$$

in which n_{eh0} and n_{el0} are the equilibrium number densities of electron species at lower and higher temperatures T_{el} and T_{eh} respectively; z_d is the number of electronic charge attached to the dust particles; $\mu_d = m_d/m_i$ in which m_i and m_d are respectively the masses of ions and dust particles; ϕ is the electrostatic potential. In (1)-(5) all the velocities are normalized by ion-acoustic speed $(k_B T_{eff} / m_i)^{1/2}$, k_B being the Boltzmann constant; all densities are normalized by equilibrium ion density n_{i0} ; potential ϕ is normalized by $k_B T_{eff} / e$; all lengths by Debye length $\sqrt{\frac{\epsilon_0 T_{eff}}{n_0 e^2}}$ and time by the inverse of ion plasma frequency $\sqrt{\frac{n_0 e^2}{\epsilon_0 m_i}}$. The charge neutrality condition is given by,

$$n_{el0} + n_{eh0} + n_{d0} z_d = 1 \quad (7)$$

in which n_{d0} is the equilibrium dust particle number density normalized by the background ion number density n_{i0} .

For nonisothermal electrons the number densities n_{el} and n_{eh} are assumed to be given by [23], [25]:

$$n_{el} = n_{el0} \left[1 + \frac{\phi}{\beta_l} - \frac{4}{3} b_{l1} \left(\frac{\phi}{\beta_l}\right)^2 + \frac{1}{2} \left(\frac{\phi}{\beta_l}\right)^2 - \frac{8}{15} b_{l2} \left(\frac{\phi}{\beta_l}\right)^5 + \frac{1}{6} \left(\frac{\phi}{\beta_l}\right)^3 - \dots \right] \quad (8)$$

$$n_{eh} = n_{eh0} \left[1 + \frac{\phi}{\beta_h} - \frac{4}{3} b_{h1} \left(\frac{\phi}{\beta_h}\right)^2 + \frac{1}{2} \left(\frac{\phi}{\beta_h}\right)^2 - \frac{8}{15} b_{h2} \left(\frac{\phi}{\beta_h}\right)^5 + \frac{1}{6} \left(\frac{\phi}{\beta_h}\right)^3 - \dots \right] \quad (9)$$

where,

$$\beta_l = T_{el}/T_{eff}, \beta_h = T_{eh}/T_{eff}, \quad b_{l1} = \frac{1-\beta_l}{\sqrt{\pi}}, b_{h1} = \frac{1-\beta_h}{\sqrt{\pi}}, b_{l2} = \frac{1-\beta_l^2}{\sqrt{\pi}}, b_{h2} = \frac{1-\beta_h^2}{\sqrt{\pi}} \quad (10)$$

Note that the parameters $b_{11,h}$ and $b_{21,h}$ measure nonisothermality in plasma. From definition it is clear that $\beta_l < 1 < \beta_h$ for all values of n_{e10} and n_{eh0} .

III. NONLINEAR ANALYSIS AND DOUBLE LAYER SOLUTIONS

To investigate the nonlinear propagation of ion-acoustic waves we employ the pseudo potential technique and assume all the dependent variables to depend only on a single variable

$$\xi = x - Mt \quad (11)$$

where M is the velocity of the nonlinear structure normalized by the ion-acoustic speed. We also use the steady state condition and impose the boundary conditions

$$\phi \rightarrow 0, n_i \rightarrow 1, n_d \rightarrow n_{d0}, u_i \rightarrow 0, u_d \rightarrow 0 \text{ as } |\xi| \rightarrow \infty \quad (12)$$

Using the transformation (11) in (1)-(4) and then integrating the resulting equations with the boundary conditions (12), we get

$$n_i = \frac{1}{2\sqrt{3\sigma_i}} \left[\sqrt{(A+B-2\phi)} - \sqrt{(A-B-2\phi)} \right] \quad (13)$$

$$n_d = \frac{n_{d0}}{\sqrt{1 + \frac{2z_d}{\mu_d M^2} \phi}} \quad (14)$$

where

$$A = M^2 + 3\sigma_i \text{ and } B = \sqrt{12\sigma_i M^2} \quad (15)$$

Now using the transformation (11), (13) and (14) in the Poisson's equation (5) and then making binomial expansions under the assumption $|\phi| < 1$ we derive

$$\frac{d^2 \phi}{d\xi^2} = S_1 \phi + S_2 \phi^{\frac{3}{2}} + S_3 \phi^2 + S_4 \phi^{\frac{5}{2}} \dots \quad (16)$$

where,

$$S_1 = \frac{n_{e10}}{\beta_l} + \frac{n_{eh0}}{\beta_h} - \frac{n_{d0} z_d^2}{\mu_d M^2} - \frac{1}{2\sqrt{3\sigma_i}} \left[\frac{1}{\sqrt{A-B}} - \frac{1}{\sqrt{A+B}} \right] \quad (17)$$

$$S_2 = -\frac{4}{3} b_{1i} n_{e10} \left(\frac{1}{\beta_l} \right)^{\frac{3}{2}} - \frac{4}{3} b_{1h} n_{eh0} \left(\frac{1}{\beta_h} \right)^{\frac{3}{2}} \quad (18)$$

$$S_3 = \frac{1}{2} n_{e10} \left(\frac{1}{\beta_l} \right)^2 + \frac{1}{2} n_{eh0} \left(\frac{1}{\beta_h} \right)^2 + \frac{3}{2} \frac{n_{d0} z_d^3}{\mu_d^2 M^4} - \frac{1}{4\sqrt{3\sigma_i}} \left[\frac{1}{\sqrt{(A-B)^3}} - \frac{1}{\sqrt{(A+B)^3}} \right] \quad (19)$$

$$S_4 = -\frac{8}{15} n_{e10} b_{2l} \left(\frac{1}{\beta_l} \right)^{\frac{5}{2}} - \frac{8}{15} n_{eh0} b_{2h} \left(\frac{1}{\beta_h} \right)^{\frac{5}{2}} \quad (20)$$

To obtain the double layer solution of (16) in the lowest order, we retain terms up to ϕ^2 and then integrating it we get

$$\frac{1}{2} \left(\frac{d\phi}{d\xi} \right)^2 + \psi(\phi) = 0 \quad (21)$$

where

$$\psi(\phi) = -\frac{S_1 \phi^2}{2} - \frac{2S_2 \phi^{\frac{5}{2}}}{5} - \frac{S_3 \phi^3}{3} \quad (22)$$

is known as Sagdeev potential [26].

The nonintegral powers of ϕ can be made integral by the substitution $\phi = y^2$ which changes the energy equation (21) to

$$\frac{1}{2} \left(\frac{dy}{d\xi} \right)^2 + \psi(y) = 0 \quad (23)$$

where,

$$\psi(y) = -\frac{S_1}{8} y^2 - \frac{S_2}{10} y^3 - \frac{S_3}{12} y^4 \quad (24)$$

For the double layer solution the modified Sagdeev potential $\psi(y)$ must satisfy the following conditions

- i. $\psi(y) = 0$ at $y = 0, y_m$
 - ii. $\psi'(y) = 0$ at $y = 0, y_m$
 - iii. $\psi''(y) < 0$ at $y = 0, y_m$
- (25)

where y_m is the amplitude of the modified double layer.

Applying the conditions (25) we get,

$$y_m = -\frac{3S_2}{5S_3}, S_1 = \frac{2S_3}{3} y_m^2 \text{ and } S_2 = -\frac{5S_3}{3} y_m \quad (26)$$

Thus for double layer solution the coefficients S_1, S_2 and S_3 must satisfy the compatibility condition

$$6S_2^2 - 25S_1 S_3 = 0 \quad (27)$$

Then (24) can be rewritten as

$$\frac{dy}{d\xi} = \pm \sqrt{\frac{S_3}{6}} y (y_m - y) \quad (28)$$

The double layer solution of (28) is

$$y = \frac{y_m}{2} \left[1 - \tanh \left(\frac{y_m \xi}{\sqrt{24/S_3}} \right) \right] \quad (29)$$

Returning to the original variable we get

$$\phi = \frac{1}{4} \phi_m [1 - \tanh(\xi/W)]^2 \quad (30)$$

in which $\phi_m = y_m^2$ is the amplitude and $W = \sqrt{24/S_3\phi_m}$ is the thickness of the double layer. Note that the double layer solution exists only in the parametric region for which the coefficient of the cubic nonlinear term in the Sagdeev potential (22) is positive i.e. $S_3 > 0$.

In order to study the effects of higher order nonlinearity and nonisothermality now we retain terms up to $\phi^{5/2}$ in (16) and then integrating we get

$$\left(\frac{d\phi}{d\xi}\right)^2 = A_1\phi^2 - A_2\phi^{5/2} - A_3\phi^3 - A_4\phi^{7/2} \quad (31)$$

where,

$$A_1 = S_1, A_2 = -\frac{4}{5}S_2, A_3 = -\frac{2}{3}S_3 \text{ and } A_4 = -\frac{4}{7}S_4$$

Note that the coefficients A_2 and A_4 include the effects of nonisothermality of electrons while A_1 and A_3 include the effects of charged dust grains. In order to find double layer solution of (31) we introduce as before a transformation $\phi(\xi) = y^2(\xi)$ and thereafter adjusting the nonlinear coefficients we can write (31) as

$$\frac{dy}{d\xi} = \pm\sqrt{\alpha}y(y_0 - y)^{3/2} \quad (32)$$

where

$$\alpha = A_4/4, y_0 = -A_3/3A_4 \text{ and } A_3^3 = -27A_2A_4^2 \quad (33)$$

In terms of S_1, S_2, S_3 and S_4 the conditions (33) can be written as

$$35S_3^2 = 108S_2S_4 \text{ and } 14S_2S_3 = 135S_1S_4 \quad (34)$$

In order to solve (32) we make the variable substitution $z = \sqrt{y_0 - y}$ and then separating the variables and integrating we obtain:

$$\xi = \pm \frac{1}{y_0\sqrt{\alpha}} \left[\frac{1}{\sqrt{y_0}} \ln \frac{\sqrt{y_0} + \sqrt{y_0 - y}}{\sqrt{y_0} - \sqrt{y_0 - y}} - \frac{2}{\sqrt{y_0 - y}} \right] \quad (35)$$

Returning to the original variable we can express ϕ as an implicit function of ξ :

$$\xi = \pm \frac{1}{y_0\sqrt{\alpha}} \left[\frac{1}{\sqrt{y_0}} \ln \frac{\sqrt{y_0} + \sqrt{y_0 - \sqrt{\phi}}}{\sqrt{y_0} - \sqrt{y_0 - \sqrt{\phi}}} - \frac{2}{\sqrt{y_0 - \sqrt{\phi}}} \right] \quad (36)$$

IV. RESULTS AND DISCUSSIONS

In the present paper we have theoretically investigated the formation of ion-acoustic double layers in a plasma consist of two-temperature nonisothermal electrons, negatively charged dust grains and warm positive ions. The double layer solution (30) includes lowest order nonlinear and nonisothermal effects. To study the effects of different plasma parameters on the shape of double layer we have plotted double layer profile given by (30) for typical plasma parameters. Charged dust grains and nonisothermal electrons are found to affect the structure of the double layer significantly.

In the model plasma under consideration, only compressive double layer is obtained.

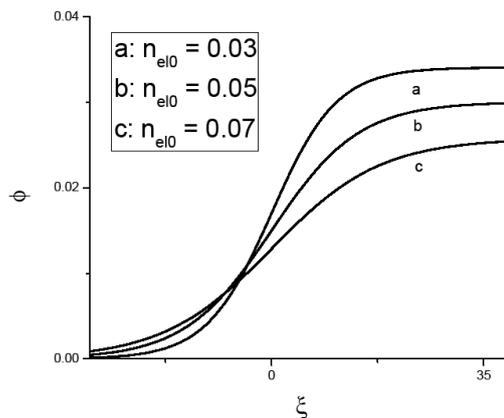


Fig 1 Double layer profile for different values of cold temperature electron concentration ($n_{e10} = 0.03, 0.05$ and 0.07) with ion-temperature $\sigma_i = 0.01$, Mach number = 1.5, dust charge number = 1, $n_{d0} = 0.1, \mu_d = 10^3, \beta_1 = 0.6, \beta_h = 1.2$

When the lower temperature electrons are present in small numbers the amplitude of double layer decreases with increase in the percentage of lower temperature electrons (Fig. 1) where as when the lower temperature electrons are present in large numbers, the amplitude of double layer increased with increase in the percentage of lower temperature electrons (Fig. 2).

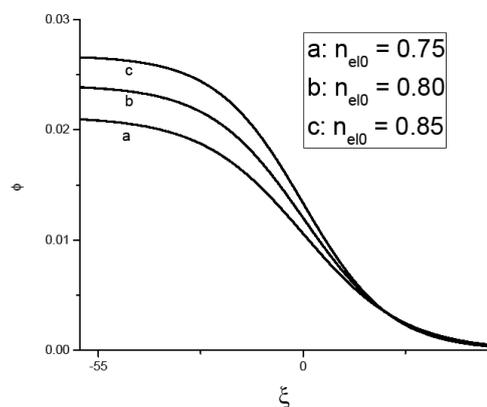


Fig 2 Double layer profile for different values of cold temperature electron concentration ($n_{e10} = 0.75, 0.8$ and 0.85) with ion-temperature $\sigma_i = 0.01$, Mach number = 1.5, dust charge number = 1, $n_{d0} = 0.1, \mu_d = 10^3, \beta_1 = 0.6, \beta_h = 1.2$

From Fig. 2 we can see that there exists a critical value of lower temperature electron concentration at which the electric field inside the double layer changes its direction. Double layer profiles for various values of ion temperature are shown in Fig. 3. Obviously the double layer amplitude decreases with increase in ion temperature.

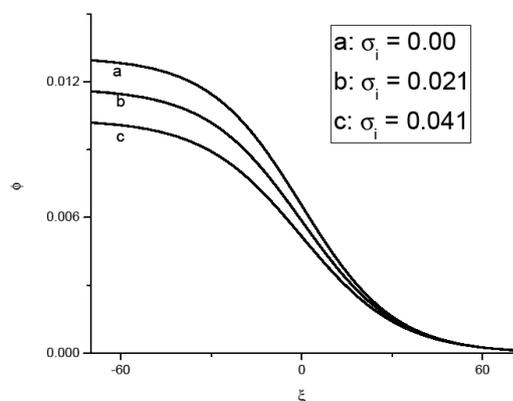


Fig 3 Double layer profile for different values of ion temperature ($\sigma_i = 0.00, 0.021$ and 0.041); Mach number = 1.2, dust charge number = 1, $n_{d0} = 0.4$, $\mu_d = 10^3$, $\beta_l = 0.6$, $\beta_h = 1.2$, $n_{e10} = 0.7$

Numerically we have also studied the effects of charged dust concentration and ion temperature on double layer width (Fig. 4).

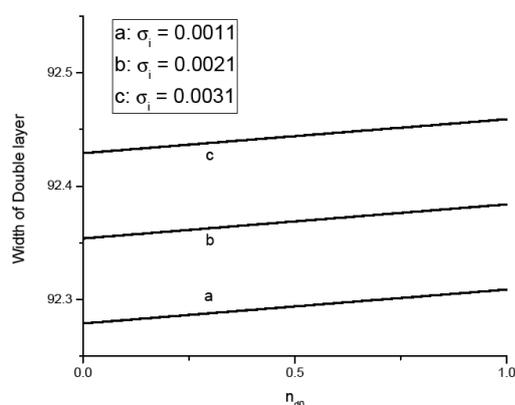


Fig 4 Double layer width plotted as a function of dust concentration for different ion temperature ($\sigma_i = 0.0011, 0.0021$ and 0.0031) with Mach number = 1, dust charge number = 1, $n_{e10} = 0.75$, $\mu_d = 50$, $\beta_l = 0.6$, $\beta_h = 1.2$

From Fig. 4 we find that double layer width increases with increase in charged dust concentration or ion temperature.

We have also studied the effects of higher order nonlinearity and nonisothermality on double layer structure. We note that the behavior of double layer solution (36) including higher order nonlinearity and nonisothermality is quite different from the weak double layer solution (30). However we have not tried to realize numerically double solution of higher order. In fact it requires two conditions (34) to be satisfied simultaneously among the coefficients S_1, S_2, S_3 and S_4 which is very stringent. The type of plasma

described in this paper can be obtained in an experimental installation of a double plasma device by RF heating of the plasma as suggested by [25]. The degree of nonisothermality can be adjusted by controlling the experimental plasma parameters from outside and the desired features of different double layer structures can be observed.

Finally we would like to point out that the results presented in this paper may be helpful in understanding the acceleration of charged particles to high energies in space and astrophysical plasmas. The investigation could be generalized to more realistic environment by including dust charge variation, inhomogeneity of the plasma and the effects of magnetic field which is beyond the scope of the present work.

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