A Hybrid Particle Swarm Optimization-Nelder-Mead Algorithm (PSO-NM) for Nelson-Siegel-Svensson Calibration

Sofia Ayouche, Rachid Ellaia, Rajae Aboulaich

Abstract—Today, insurers may use the yield curve as an indicator evaluation of the profit or the performance of their portfolios; therefore, they modeled it by one class of model that has the ability to fit and forecast the future term structure of interest rates. This class of model is the Nelson-Siegel-Svensson model. Unfortunately, many authors have reported a lot of difficulties when they want to calibrate the model because the optimization problem is not convex and has multiple local optima. In this context, we implement a hybrid Particle Swarm optimization and Nelder Mead algorithm in order to minimize by least squares method, the difference between the zero-coupon curve and the NSS curve.

Keywords—Optimization, zero-coupon curve, Nelson-Siegel-Svensson, Particle Swarm Optimization, Nelder-Mead Algorithm.

I. INTRODUCTION

ANY investor is exposed to rate risk, and he should anticipate rate movements in the future. Constructing a term structure of interest rates most often refers to the concept of zero-coupon. The zero coupon curves are calculated through the yield curves of the market and are used mainly for evaluating financial contracts. The calibration of the zero coupon curve consists in reconstructing the yield curve using data observed in the market. This reconstruction is necessary due to the fact that there is not enough zero-coupon bonds (strips) listed on the market. In addition, zero coupon bonds often have less liquidity than coupon bonds. In this context, various curve fitting methods have been introduced. The most popular approaches to the term structure modeling are various curve fitting spline methods initiated by McCulloch [12] and who fit a cubic Spline to the discount curve and also Vasicek and Fong [15] who model the discount curve with exponential Spline. Indeed Bliss and Fama [1] developed an iterative method for fitting the forward rate curves, sometimes called unsmoothed Fama-Bliss. Hence, these methods have been criticized for not having economic properties. Therefore, Nelson and Siegel [13] and Svensson [14] proposed parametric curves that are flexible enough to describe the whole family of observed term structure shapes. Despite the absence of the no-arbitrage restriction, the function developed by Nelson Siegel and its augmented version by Svensson matches the yield curve quite well and is widely used by many central banks for yield curve modeling.

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In this paper, we restrict ourselves to the Nelson-Siegle-Svensson model in order to reconstruct and forecast the term structure of interest rates, and this can help the investor to better manage a portfolio of products rate. Many authors founded a lot of difficulties in calibrating the model since the function is not convex, especially those who use methods based on derivatives of the objective function. Gilli, Grosse and Schumann [5] analyze the calibration of the model and implement and test an optimization heuristic, Differential Evolution, to obtain parameters. Differential Evolution gives solutions that fit the data very well. Gimeno et al. [6] proposed the use of genetic algorithms as an alternative optimization methodology to the traditional methods. They find better results than traditional methods.

We use the hybrid particle swarm optimization and Nelder Mead algorithm to calibrate the model using Moroccan Government bonds. We presented a modified PSO using a direct search complex algorithm to control the PSO heuristic parameters. The paper is structured as follows: Section II discusses in detail the calibration of the NSS model using heuristic optimization methods. Section III concludes and presents new perspectives on the basis of work done.

II. CALIBRATING THE NELSON-SIEGLE-SVENSSON MODEL TO CONSTRUCT THE YIELD CURVE

A. Description of the NSS Model

The Nelson-Siegel [13] and its extension developed by Svensson [14] is a parametric model, which was designed to describe the movement of the entire range of rates and to reconstruct the yield curve. Moreover, it is a dynamic method that uses parameters changing in time. These parameters are estimated to a high level of accuracy and are considered as factors that match the level, slope and the interest rate curve of government bonds. Unlike the model of Nelson-Siegel, the Svensson can fit different shapes of yield curve that can be found on the market, especially curve with a bump or a hollow. See for instance [4]. The resulting Nelson-Siegel approximating forward curve can be assumed to be the solution to a second order differential equation with equal roots for spot rates

$$f_t(\tau, \beta) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau_1}\right) + \tag{1}$$

$$\beta_2 \frac{t}{\tau_1} \exp\left(-\frac{t}{\tau_1}\right) + \beta_3 \frac{t}{\tau_2} \exp\left(-t/\tau_2\right)$$

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The rate $NSS_t(\tau, \beta)$ of maturity t is calculated from

$$NSS_t(\tau, \beta) = \beta_0 + \beta_1 \Big(\frac{1 - \exp(-\frac{t}{\tau_1})}{\frac{t}{\tau_1}} \Big) + \qquad (2)$$

$$\beta_2 \Big(\frac{1 - \exp(-\frac{t}{\tau_1})}{\frac{t}{\tau_1}} - \exp(-\frac{t}{\tau_1}) \Big) +$$

$$\beta_3 \Big(\frac{1 - \exp(-\frac{t}{\tau_2})}{\frac{t}{\tau_2}} - \exp(-\frac{t}{\tau_2}) \Big)$$

with the following notations

$$
\begin{array}{ll}
\beta_0 : & \text{factor level} \\
\beta_1 : & \text{slope factor} \\
\beta_2 : & \text{first curvature factor} \\
\beta_3 : & \text{second curvature factor} \\
\tau_1 : & \text{first scale factor} \\
\tau_2 : & \text{second scale factor .}
\end{array}
\qquad (3)
$$

This model defines following constraints

$$
\begin{cases}
\beta_0 & \geq 0 \\
\beta_0 + \beta_1 & \geq 0 \\
\tau_1 & \geq 0 \\
\tau_2 & \geq 0
\end{cases}
\qquad (4)
$$

In the rest of this paper we denote the Nelson-Siegel[13] and its extension developed by Svensson[14] by NSS model.

1) Interpretation of the Model Parameters: The NSS model is consistent with having economic factors: Level, slope, two curvatures and two scales. Although these parameters are significant since each parameter has a particular influence on the behavior of the curve; they are also parsimonious in the sense that a small number of parameters are used to represent the curve. According to Diabold and Li [3], these parameters explain a range of term structure shapes and their dynamics. Hautsch and Ou [7] demonstrate that the factors of NSS model and their volatilities are linked to macroeconomic variables and their variances. Diebold et al.[4] examine the macroeconomic variables that affect the term structure. Their empirical work shows that the GDP growth rate and the unemployment rate are the preachers of the relative risk of the level and the slope of the yield curve of the US government Bond. This model consists of four parts reflecting six factors: β_0, β_1, β_2, β_3, τ_1 and τ_2 .

$- \beta_0$ is a constant that represents the (long term) interest rate level, hence the name "factor level". So it is the limit of the functional NSS model when maturity tends to infinity.

$$\lim_{t \to +\infty} NSS_t(\tau, \beta) = \beta_0 \qquad (5)$$

$- \beta_1$: Economically β_1 is interpreted as the difference between short rates and long rates. β_1 is positive for small maturities, but, decreases exponentially to 0 when maturity increased.

$$\lim_{t \to 0} NSS_t(\tau, \beta) = \beta_0 + \beta_1 \qquad (6)$$

$- \beta_2$ represents the first curvature of the curve, for short

maturities. The functional NSS model generates a hump for $\beta_2 > 0$ or a trough for $\beta_2 < 0$.

$- \beta_3$: β_3 as β_2, influences the amplitude and direction of the second curvature, for long maturities.

$- \tau_1$ determines the exact position of the maximum or minimum of the curve. It must be positive, because it is homogeneous in time.

$- \tau_2$ specifies the position of the second curvature.

Furthermore, $r = \beta_0 + \beta_1 = \beta_0 - (-\beta_1)$ is the instantaneous short rate. The long-term rates are more persistent and less volatile than short rates. The long-term rate depends only on β_0 while the short-term rate depends on β_0 and β_1. To resume, this model defines following constraints

$$
\begin{cases}
\beta_0 & \geq 0 \\
\beta_0 + \beta_1 & \geq 0 \\
\tau_1 & \geq 0 \\
\tau_2 & \geq 0
\end{cases}
\qquad (7)
$$

B. Methodology of Calibration

In this section, we deal with the optimization problem, so our aim will be to estimate six factors of the model, by minimizing the difference between the NSS curve and the zero-coupon curve, using least square method.

$$
\begin{cases}
\text{Minimize } f(\tau, \beta) = \sum_{i \in I} \left(NSS_t(\tau_i) - ZC_t(\tau_i) \right)^2 \\
\text{subject to:} \\
\beta_0 \geq 0 \\
\beta_0 + \beta_1 \geq 0 \\
\tau_1 \geq 0 \\
\tau_2 \geq 0
\end{cases}
\qquad (8)
$$

where
- $NSS_t(t_i)$ is the rate calculated by the NSS model, for maturity τ_i at time t.
- $ZC_t(\tau_i)$ is the observed rate, for maturity τ_i at time t.
- I are all rates available in the market at time t.

As we can see, the optimization problem is based mainly on the construction of the zero-coupon curve. To construct the zero-coupon curve, we need to convert currency rates in actuarial rates, because they are already considered as zero-coupon rates, then we transform actuarial rates to zero-coupon rates by using the linear interpolation method and the bootstrapping method.

1) Linear Interpolation Method: We consider the rate $r(t, T_1)$ of maturity T_1, and the rate $r(t, T_2)$ of maturity T_2, and we want to know the rate $r(t, T)$ of maturity T, with $T_1 < T < T_2$. For this, we use the following linear interpolation formula:

$$r(t, T) = \frac{(T_2 - T)r(t, T_1) + (T - T_1)r(t, T_2)}{T_2 - T_1} \qquad (9)$$

2) Bootstrapping Method: Bootstrapping consists to reconstruct a zero-coupon curve in spot, segment by segment maturity.
- Method of Calculating the Short Term

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For currency rates T_m of maturities $T < 365$, we just need to transform them into actuarial rates T_a using:

$$T_a = \left(1 + \frac{T_m * Mat}{360}\right)^{\frac{365}{Mat}} - 1 \qquad (10)$$

- Method of Calculating the Long Term

To determine the zero-coupon rate of maturities $T > 365$, we accept the hypothesis that the theoretical price of a bond is the sum of its cash flows discounted at the zero-coupon rate, for each refund.

$$P = \frac{c}{1+a} + \frac{c}{(1+a)^2} + \cdots \frac{c}{(1+a)^n} + \frac{N}{(1+a)^n}$$
$$= \frac{c}{1+ZC_1} + \frac{c}{(1+ZC_2)^2} + \cdots \frac{c}{(1+ZC_n)^n} + \frac{N}{(1+ZC_n)^n} \qquad (11)$$

Moreover, we assume that nominal rates are equal to actuarial rates, so, the nominal interest rate T_f for maturity i is the rate of return T_r of the same maturity.

$$\begin{array}{lll}
\text{for 1 year :} & T_f = T_r(1 \text{ year}) \\
\text{for 2 years :} & T_f = T_r(2 \text{ years}) \\
\ldots & \ldots \\
\text{for n years :} & T_f = T_r(\text{n years})
\end{array} \qquad (12)$$

$$\text{so :} \qquad ZC_1 = T_r(1 \text{ year})$$

We deduce ZC_2(2 years) from the following form of equality

$$P = \frac{c}{1+a} + \frac{c+N}{(1+a)^2} = \frac{c}{1+ZC_1} + \frac{c+N}{(1+ZC_2)^2}. \qquad (13)$$

giving

$$ZC_2 = \left(\frac{1+Tr_2}{1-\dfrac{Tr_2}{1+ZC_1}}\right)^{\frac{1}{2}} - 1, \qquad (14)$$

we continue by iterative process to determine the "zero-coupon rate" of other maturities, and we also use the linear interpolation method on the actuarial curve. So, for n years, we find

$$ZC_n = \left(\frac{1+Tr_n}{1-\displaystyle\sum_{i=1}^{n-1}\dfrac{Tr_n}{1+ZC_i}}\right)^{\frac{1}{n}} - 1. \qquad (15)$$

After constructing the zero coupon curve, we are now able to estimate NSS parameters. As we can see, we are dealing with a nonlinear and no convex least-square problem featuring a well-defined objective. The functional implementation, although linear in slope factors and curvature, is not compared to the scale factors. Hence, the nonlinear model structure seems to pose serious difficulties for optimization procedures to arrive at reasonable estimates. This difficulty stems from the coexistence of local minimum near the global minimum. Furthermore, we would like to fit a term

structure with some reasonable degree of smoothness. So, it is preferable to choose appropriate procedures of optimization that provide us with a good, possibly near-optimal, solution. There are different approaches to overcome this problem. An example is shown and discussed in Bonnin et al. [2] for the Svensson model. They solve this problem using linear regressions and they address numerical problems and estimation issues when estimating the Svensson model. Gilli et al. [5] run 500 times a gradient search algorithm using different parameter values of the model. They see a strong instability of the parameters estimated, although the resulting curve fits with precision the yield curve observed at the market. Here we chose to apply meta-heuristic methods to solve this problem. More specifically we chose to apply the hybrid Particle Swarm Optimization (PSO) and Nelder-Mead algorithm for two main reasons:
- The objective function is multimodal hence the importance of a global search method.
- Nelder- Mead algorithm improves much more the result found with the optimization of PSO.

3) Particle Swarm Optimization: Particle Swarm Optimization algorithm is a member of the wide category of swarm intelligence methods which are an alternative class of stochastic search algorithms to solve non-linear programming problems. It was proposed by Kennedy and Eberhart [9]. In PSO algorithm, the individual is called particle which has no mass and volume. The particles of the swarm are randomly arranged in the search space and each particle is associated with a velocity, which is constantly updated by the particle's previous best performance and by the previous best performance of the particle's neighbors. The next iteration takes place after all particles have been moved. In this iteration, each member of the swarm updates its position and its velocity, and the objective function is recalculated for each individual, which help to determine the minimum of the problem, before reaching the stopping criterion. See for instance article [9]. The principle of the PSO algorithm relies on 3 components, they are:
- *The current position of the particle* represents the coordinates of the particle in the search space.
- *The best position of the particle is the value* that represents the best solution of the particle.
- *The speed of movement of a given particle* determines the next position of this particle.
The particles are manipulated according to the following vectorial equations:

$$\begin{cases}
V_{t+1} = & \omega_t.V_t + C_1.rand()(pbest - X_t) + \\
& C_2.rand()(vbest - X_t) \\
X_{t+1} = & V_{t+1} + X_t
\end{cases} \qquad (16)$$

- $X_i(t)$: Its position in the search space.
- $V_i(t)$: Its velocity.
- X_{pbest_i}: The position of the best solution of the particle.
- X_{vbest_i}: The position of the best solution of its neighborhood.
- $pbest_i$: The value of the fitness of its best solution.

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- $vbest_i$: The value of the fitness of the best solution in the neighborhood.
- ω: An inertia factor, defines the exploration capacity of each particle.
- C_1, C_2: Constants, they are considered as accelerators.

4) Nelder-Mead Algorithm: Nelder-Mead algorithm, also called downhill Simplex is a deterministic search strategy relying on functions evaluations only. It is an optimization method for non-linear problems, which uses a simplex of $n+1$ points.

The principle of the algorithm consists to evaluate the objective function by moving the simplex until a minimum is found. The operations of this method are to rescale the simplex based on the local behavior of the function by using three basic procedures: reflection, expansion and contraction. Although the simplex search procedure has its merit, it does not overcome the possible difficulties due to the non-convexity of the objective function.

5) The Hybrid PSO-Nelder Mead Algorithm: PSO-NM optimization method integrates evolutionary algorithm (PSO) and traditional algorithm (Nelder Mead algorithm). In recent years, many authors have reported this hybrid [11], [10]. The PSO optimization finds the local best solution easily, but it requires many particles in an optimal process, which reduces the speed of computation. Combining the two algorithms has been a very popular approach to speed up convergence. The NM-PSO algorithm enables feasible optimal solutions that satisfy the constraint conditions [8]. At first, we applied the global minimization algorithm PSO, then we introduce Downhill Simplex for the problem (8). The algorithm that we have adopted for solving this problem is:

```
              ***PSO algorithm***
1: Initialization
Initialize positions X_i and velocities V_i of each particle.
2: Assignment
P_i = X_i
3: Iteration
as the stopping test is not verified, do:
        V_i^{iter+1} = wV_i^{iter} + c_1 r_1^{iter}(P_b^{iter} - X_i^{iter}) +
        c_2 r_2^{iter}(P_g^{iter} - X_i^{iter})
        X_i^{iter+1} = X_i^{iter} + V_i^{iter}
4: Evaluation
        if f(X_i) < f(P_i)
        P_i = X_i
End if
End as
5: Return X*

        *** Nelder-Mead algorithm ***
6: Calculating f(X_0) with X_0 = X*
7: Evaluation
        X_r = X_0 + (X_0 - X_{N+1})
        if f(X_r) < f(X_N)
        X_e = X_0 + 2(X_0 - X_{N+1})
        if f(X_e) < f(X_r)
        X_{N+1} = X_e
        else X_{N+1} = X_r, return to step 6.
        if f(X_N) < f(X_r)
        X_c = X_{N+1} = 1/2(X_0 - X_{N+1})
        if f(X_c) < f(X_N)
        X_{N+1} = X_c ,return to step 6.
        else X_i = X_1 + 1/2(X_0 - X_1), return to step 6.
8: Return X**
```
(17)

With the following notations:

$$
\begin{aligned}
X &: && \text{a particle } (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2) \\
X^* &: && \text{optimum returned by the PSO method} \\
X^{**} &: && \text{optimum returned by the hybrid algorithm} \\
P_g &: && \text{position of the global particle} \\
P_b &: && \text{best position of the particle} \\
c_1, c_2 &: && \text{random factors, accelerators} \\
\omega &: && \text{inertia} \\
Xr &: && \text{reflection point of the simplex} \\
Xc &: && \text{contraction point of the simplex} \\
Xe &: && \text{expansion point of the simplex}
\end{aligned}
$$
(18)

The Nelder-Mead algorithm is defined as an unconstrained non-linear mathematical programming problem, so, in order to satisfy the constraints of our problem (8), we need to penalize the objective function, by introducing a penalty constant (λ), as following:

$$
\begin{cases}
f(X) = & \sum_{i \in I} (NSS_t(\tau_i) - ZC_t(\tau_i))^2 + \\
& \lambda([C_1(X)]^- [C_2(X)]^- + [C_3(X)]^- + \\
& [C_4(X)]^-) \\
\text{with} \\
[C_1(X)]^- = & max\,(0, -\beta_0) \\
[C_2(X)]^- = & max\,(0, -\beta_0 - \beta_1) \\
[C_3(X)]^- = & max\,(0, -\tau_1) \\
[C_4(X)]^- = & max\,(0, -\tau_2)
\end{cases}
$$
(19)

with λ is a penalty parameter which varies between 50 and 500.

C. Results

The Moroccan government issues bonds called Treasury Bills to cover its financing needs and repay its debt. Bank Al Maghrib regularly publishes a yield curve taking into account the operations of most primary and secondary recent markets. The published rates are, for every term, average rates balanced by the prices. Yields are expressed in currency rates for maturities $\leq 1year$ and in actuarial rates for maturities $\geq 1year$. The calculation of interest rates include interpolation and transformation rates mentioned in Sections $B1$ and 2. So, to illustrate our Nelson-Siegel term structure fitting procedure we use Moroccan treasury bills. We consider the Morrocan yield curve issued on $28/04/2014$, like an example, to simulate our program calibration. For this date, the calibration algorithm gives the following parameters:

TABLE I
NSS PARAMETERS FOR THE DAY 28/04/2014

β_0	β_1	β_2	β_3
0.1201775	-0.088363897	-0.037750091	-0.1384461
τ_1	τ_2		
2.999999902	9.916963315		

The program of calibration has been validated for an error about 10^{-4}.[1]
The following table provides a comparison between the NSS curve and the zero-coupon curve.

[1]The error represents the absolute difference between the NSS rate and the ZC rate.

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TABLE II
COMPARISON BETWEEN NSS RATES AND ZC RATES FOR THE DAY
28/04/2014

Maturity(in year)	ZC rate	NSS rate	Error*10^4
0.0301	3.191%	3.186%	0.567
0.2219	3.182%	3.215%	3.302
0.4684	3.216%	3.254%	3.835
0.4986	3.250%	3.259%	0.879
0.6136	3.275%	3.278%	0.318
0.8027	3.336%	3.310%	2.595
1	3.363%	3.345%	1.824
2	3.536%	3.529%	0.627
3	3.730%	3.719%	1.101
4	3.894%	3.904%	1.006
5	4.059%	4.081%	2.225
6	4.227%	4.249%	2.181
7	4.399%	4.407%	0.791
8	4.576%	4.557%	1.915
9	4.757%	4.698%	5.880
10	4.831%	4.834%	0.252
11	4.938%	4.964%	2.565
12	5.066%	5.090%	2.441
13	5.198%	5.213%	1.542
14	5.335%	5.334%	0.109

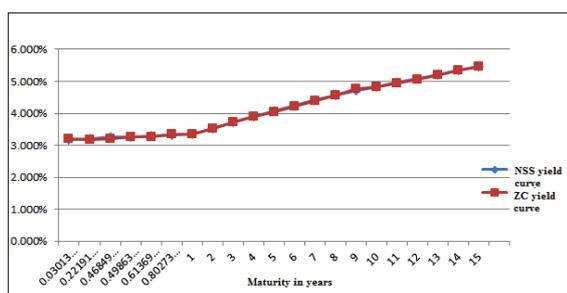Results indicated at Table II were plotted as Fig. 1.

Fig. 1 Comparison between NSS curve and ZC curve for the day
28/04/2014

In Fig. 1, we clearly see that the "NSS yield curve" fits the "ZC yield curve". As a result, the NSS curve in blue, does not seem because it is adjusted under the zero-coupon curve, and this on the interval of maturities going from 1 day to 15 years. The calibration algorithm has been validated for 580 yield curve which 83% are fitted to an average error is about 5.10^{-4}, and 17% with a maximum average error is about 9.10^{-4}.

We chose to do the calibration for the curve over the range of maturities from 1 day up to 15 years for two main reasons:

- Moroccan curve generally represents actuarial rates for maturities $\leq 25 years$, so a linear interpolation to construct zero-coupon curve for a maturity of more than this limit will not exist. In this case, our program returns an exception saying that the interpolation is not allowed, and extrapolation is not recommended because it will be uncertain rates.
- The BAM curve shows bumps and troughs particularly for long term rate (maturity $\geq 15 years$). However, it loses accuracy during calibration.

III. CONCLUSION

In this paper, we have analyzed the calibration of the Nelson-Siegel-Svensson model to construct and forecast the Moroccan yield curve. This model is widely used, but it is rarely discussed because that fitting often causes problems, specially, if we do not use heuristic optimization methods. We proposed the hybrid between Particle Swarm Optimization and Nelder-Mead algorithm as an alternative optimization methodology to the traditional methods. These heuristics optimizations gave results that were reliably better than those obtained by a traditional method based on the derivatives of the objective function.

Based on our researches and our experiment, we propose to study the dynamic of the the NSS model and to choose appropriate methods to forecast the yield curve. We propose also to more improve the calibration with Nelson-Siegel-Svensson model, and this, by controlling the volatility of the model parameters, by expressing them as stochastic processes, in order to control the volatility and liquidity problems in Moroccan market.

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