Numerical and Experimental Investigations of Cantilever Rectangular Plate Structure on Subsonic Flutter

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***Abstract*—**In this study, flutter characteristics of cantilever rectangular plate structure under incompressible flow regime are investigated by comparing the results of commercial flutter analysis program ZAERO© with wind tunnel tests conducted in Ankara Wind Tunnel (ART). A rectangular polycarbonate (PC) plate, 5x125x1000 mm in dimensions, is used for both numerical and experimental investigations. Analysis and test results are very compatible with each other. A comparison between two different solution methods (*g* and *k-method*) of ZAERO© is also done. It is seen that, *k-method* gives closer result than the other one. However, g-method results are on conservative side and it is better to use conservative results namely g-method results. Even if the modal analysis results are used for the flutter analysis for this simple structure, a modal test should be conducted in order to validate the modal analysis results to have accurate flutter analysis results for more complicated structures.

***Keywords*—**Flutter, plate, subsonic flow, wind tunnel.

I. Introduction

INTERACTION of aerodynamic, inertial and elastic forces may result in instabilities. One of the most important instability known in aeroelasticity is flutter. Flutter is an aeroelastic instability which involves one flap-wise and one torsional degrees of freedom (DoF). Coupling of the torsional structural mode with the flap-wise bending mode results in a flutter mode. Torsion of the structure is the result of the aerodynamic forces. The angle of attack is changed by the torsion. As a result of angle of attack change aerodynamic lift force is also changed [1].

The change of angle of attack due to torsion changes the lift in an unfavorable phase with the flap-wise bending which results in flutter. Vibrations grow rapidly at flutter speed. Structural damping cannot compensate the negative damping caused by the flutter mode. Flutter is observed above a certain relative wind speed on the structure, this speed is called as the critical flutter speed [2].

Design of a new air vehicle or a new external store for an existing air vehicle is a complicated design task. That design task gets more complicated since more complex systems are desired by the consumers as a result of rapid progress in the engineering technology. It is also expected to decrease the design cost of these new products since the cost is always an important design consideration. Another important design consideration is the time. There is not infinite time for research and development of a new product. MIL-HDBK-1763 "Aircraft/Stores Compatibility: Systems Engineering Data Requirements and Test Procedures" explains how to certify a newly developed military aircraft or a new external store for an existing combat aircraft. Some of the tests designated in this standard are expensive and time consuming, like Ground Vibration Testing (GVT) and flutter test. Therefore it is expected to decrease time and expenses consumed for these tests. The main objective of this work is to determine degree of accuracy of flutter analysis results of plate like structures in incompressible flow compared to wind tunnel flutter test results and also compare the *k-method* and *g-method* solution methods of ZAERO©. It is also objected to compare the different modal analysis results, in which different structural boundary conditions used, with the modal test result of the plate like structure. More accurate flutter analyses are going to decrease the number of flutter tests.

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In this study, flutter analyses were realized with ZAERO©, a commercial aeroelastic analysis software that uses panel method based on linearized potential flow theory. Modal parameters (natural frequencies and mode shapes) required by ZAERO© were obtained from MSC NASTRAN© solver. All structural FE models were constructed in MSC PATRAN©. Finally, all flutter tests were conducted in Ankara Wind Tunnel (ART).

The study of flutter begins with the research of Lanchester [3], Bairstow and Fage [4] in 1916 about the antisymmetrical flutter of a Handley Page bomber. In 1918 Blasius [5] started to make some calculations for the Albatros D3 biplane due to the failure of the lower wing of that plane. The development of the flutter analysis is increased after the development of non-stationary airfoil theory by Kutta and Joukowsky. The torsion flutter was first found by Glauret in 1929. It is discussed in detail by Smilg [6]. Several types of single degree of freedom flutter involving control surfaces at both subsonic and supersonic speeds have been found [7]-[8].

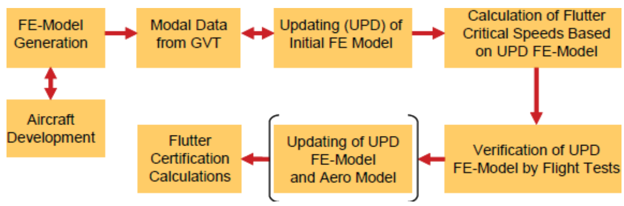

Fig. 1 Actual verification and validation process of aeroelastic aircraft models [21]

Pure bending flutter of a cantilever swept wing occurs if the wing is heavier than the surrounding air and has a sufficiently large sweep angle [9]. The bending torsion in an incompressible fluid has been studied by J.H.Greidanus [10]. Dugundji [11] searched for panel flutter and the rate of damping.

Dowell [12], [13] investigated the two and three-dimensional plate undergoing cyclic oscillations and aeroelastic instability. Cantilever beam with tip loads having an arbitrary cross section is discussed by Kosmataka [14] using a power series solution technique for the out of plane flexure and torsion case. Subsonic flight and supersonic flight is an ordinary event nowadays. Hypersonic flights become more and more popular due to increased needs. As a result, aeroelastic analyses become more important part of the design of a new aircraft or an external store for an existing aircraft.

TABLE I
NOMENCLATURE

Symbol	Meaning
$[M_{GG}]$	Generalized mass matrix generated by structural FE-model
$[K_{GG}]$	Generalized stiffness matrix generated by structural FE-model
$\{x(t)\}$	Structural deformation
$\{\ddot{x}(t)\}$	Second derivative of the structural deformation
$\{F(t)\}$	Aerodynamic forces applied on the structure
$\{F_a(t)\}$	Aerodynamic forces induced by the structural deformation
$\{F_e(t)\}$	External forces
q_∞	Aerodynamic pressure
L	Reference length
c	Reference chord length
V	Velocity of the undisturbed flow
$[\Phi]$	Truncated modal matrix
$\{q(s)\}$	Generalized coordinates, i.e. modal coordinates
$[M_{HH}]$	Generalized (modal) mass matrix
$[K]$	Generalized (modal) stiffness matrix
ω	Harmonic oscillatory frequency
$\{h\}$	Structural deformation defined at the aerodynamic boxes
$\{F_h\}$	Resultant aerodynamic forces at the aerodynamic boxes due to $\{h\}$
$\{\delta h\}$	Virtual displacement
$\{\delta x\}$	Virtual displacement
γ	Transient decay rate coefficient

Libo *et al*. [15] designed a wind tunnel test model for the flutter analysis. The model was used in a complete flutter certification procedure. GVT, model updating and flutter analysis were all done for this model. P-k solution method was used in flutter analysis. Neal *et al*. [16] worked on the design and wind tunnel analysis of a fully adaptive aircraft configuration. The goal of the study was to determine the effect of the sweep, span extension and tail extension on the aerodynamic characteristics of the model. Omar and Kurban [17] designed a free wing unmanned aerial vehicle model and tested it in low speed closed circuit wind tunnel to see the effect of the angle of attack and Reynold's number. Samikkannu [18] studied the details of fabrication, ground and wind tunnel testing of a scaled aeroelastic model of T-Tail with a flexible fuselage. Composite materials were used to obtain the required dynamics for the model during the GVT. Wind tunnel test was done in order to see the flutter characteristics of the model. Strand and Levinsky [19] conducted wind tunnel tests for a free-wing tilt-propeller V/STOL airplane model in order to see aerodynamic characteristics of the model airplane. Lift and drag curves of the airplane have been obtained as a function of propeller tilt angle and thrust coefficient.

In this study, ZAERO©, flutter analysis software based on linearized potential flow theory and developed by ZONA© Inc., is used for aeroelastic stability analysis. At this point it is necessary to explain the aeroelastic theory behind the software. Here, some theoretical background is given, in Table I the symbols used in equations and corresponding meanings are tabulated.

A. Aeroelastic Stability Equations

The equation of motion of an aeroelastic system can be stated as follows [20]:

$$[M_{GG}]\{\ddot{x}(t)\} + [K_{GG}]\{x(t)\} = \{F(t)\} \tag{1}$$

$\{F(t)\}$ consists of two parts:

$$\{F(t)\} = \{F_a(t)\} + \{F_e(t)\} \tag{2}$$

Combining Equations (1) and (2) gives:

$$[M_{GG}]\{\ddot{x}(t)\} + [K_{GG}]\{x(t)\} - \{F_a(t)\} = \{F_e(t)\} \tag{3}$$

If $\{F_a(t)\}$ is nonlinear with respect to $\{x(t)\}$, flutter analysis is performed by a time-marching procedure solving the following equation:

$$[M_{GG}]\{\ddot{x}(t)\} + [K_{GG}]\{x(t)\} - \{F_a(t)\} = 0 \tag{4}$$

with initial conditions $x(0)$ and $\dot{x}(0)$. Amplitude linearization assumption converts (4) into an eigenvalue problem for flutter analysis. In this case, the aerodynamic feedback $\{F_a(t)\}$ is related to the structural deformation $\{x(t)\}$ by means of the following convolution integral:

$$\{F_a(x)\} = \int_0^t q_\infty \left[H\left(\frac{V}{L}(t-\tau)\right)\right]\{x(\tau)\}\mathrm{d}\tau \tag{5}$$

where $\left[H\left(\frac{V}{L}(t-\tau)\right)\right]$ represents the aerodynamic transfer function, and L is defined as: $L = \frac{c}{2}$. The Laplace domain counterpart of the (5) is simply:

$$\{F_a(x(s))\} = q_\infty\left[\bar{H}\left(\frac{sL}{V}\right)\right]\{x(s)\} \tag{6}$$

(4) now can readily be transformed into the Laplace domain and results in an eigenvalue problem in terms of s given as follows:

$$\left[s^2[M_{GG}] + [K_{GG}] - q_\infty\left[\bar{H}\left(\frac{sL}{V}\right)\right]\right]\{x(s)\} = \{0\} \tag{7}$$

1. Modal Reduction Approach

Solving (7) directly is computationally costly since the FE model of an aircraft contains higher DOF. Therefore, the mass and stiffness matrices are very large in size. Modal reduction approach is used to solve this problem. Structural deformation is expressed in terms of modal coordinates as follows:

$$\{x(s)\} = [\Phi]\{q(s)\} \tag{8}$$

Substituting (8) into (7) and pre-multiplying (7) with $[\Phi]^T$ yields the following flutter equation:

$$\left[s^2[\Phi]^T[M_{GG}][\Phi] + [\Phi]^T[K_{GG}][\Phi] - q_\infty[\Phi]^T\left[\bar{H}\left(\frac{sL}{V}\right)\right][\Phi]\right]\{q(s)\}=\{0\} \tag{9}$$

(9) can be written as follows:

$$\left[s^2[M_{HH}] + [K_{HH}] - q_\infty\left[Q_{hh}\left(\frac{sL}{V}\right)\right]\right]\{q(s)\}=\{0\} \tag{10}$$

where $[M_{HH}] = [\Phi]^T[M_{GG}][\Phi]$, $[K] = [\Phi]^T[K_{GG}][\Phi]$, $\left[Q_{hh}\left(\frac{sL}{V}\right)\right] = [\Phi]^T\left[\bar{H}\left(\frac{sL}{V}\right)\right][\Phi]$is the generalized aerodynamic force matrix. The modal reduction approach provides reduces the size of the eigenvalue problem. Solving this equation is easier than that of (7). (10) is the classical flutter matrix equation.

In order to achieve that conversion it is desired to obtain aerodynamic transfer function. ZAERO obtains unsteady aerodynamics methods in the frequency domain by assuming simple harmonic motion. Obtained aerodynamic transfer function is called the Aerodynamic Influence Coefficient (AIC) Matrix.

2. Unified AIC of ZAERO©

ZONA6, ZONA7 are unsteady aerodynamics methods incorporated in ZAERO©, ZONA6 generates AIC matrices for subsonic flow regimes, ZONA7 generates AIC matrices for supersonic flow regimes. One of the fundamental aerodynamic parameter is the reduced frequency and it is defined as:

$$k = \frac{\omega L}{V} \tag{11}$$

ZAERO© uses the panel method which is based on the linearized potential flow theory to solve the integral equations.

3. Flutter Solution Techniques

a. K-Method

The classical flutter matrix equation derived in Section *A.I* is given as:

$$\left[s^2[M_{hh}] + [K_{hh}] - q_\infty\left[Q_{hh}\left(\frac{sL}{V}\right)\right]\right]\{q(s)\} = \{0\} \tag{12}$$

Unsteady aerodynamics methods are used by ZAERO© to formulate aerodynamic transfer function in frequency domain (k-domain).

$$[\bar{H}(ik)] = [G]^T[AIC(ik)][G] \tag{13}$$

The frequency domain counterpart of the classical flutter matrix equation can be obtained as follows:

$$\left[-\omega^2[M_{hh}] + [K_{hh}] - q_\infty[Q_{hh}(ik)]\right]\{q\} = \{0\} \tag{14}$$

If we add an artificial structural damping to (14), the K-method flutter equation is obtained as follows:

$$\left[-\omega^2[M_{hh}] + (1 + ig_s)[K_{hh}] - q_\infty[Q_{hh}(ik)]\right]\{q_{ik}\} = \{0\} \tag{15}$$

g_s is the added artificial structural damping.

b. g-Method

Assume an analytic function in the form of $[Q_{hh}(p)] = [Q_{hh}(g + ik)]$ in the domain of $g \geq 0$ and $g < 0$. $[Q_{hh}(p)]$ can be expanded along the imaginary axis (i.e. $g = 0$) for small g by means of damping perturbation method:

$$[Q_{hh}(p)] \approx [Q_{hh}(ik)] + g\left.\frac{\partial[Q_{hh}(p)]}{\partial g}\right|_{g=0} \quad \text{for } g \ll 1 \tag{16}$$

If $[Q_{hh}(p)]$ is analytic, it must satisfy the Cauchy-Riemann equations such that:

$$\frac{\partial[Re(Q_{hh}(p))]}{\partial g} = \frac{\partial[Im(Q_{hh}(p))]}{\partial k} \tag{17}$$

$$\frac{\partial[Im(Q_{hh}(p))]}{\partial g} = -\frac{\partial[Re(Q_{hh}(p))]}{\partial k} \tag{18}$$

Combining (17) and (18) yields the following general equation:

$$\frac{\partial[(Q_{hh}(p))]}{\partial g} = \frac{\partial[(Q_{hh}(p))]}{\partial(ik)} \tag{19}$$

Thus, the term $\left.\frac{\partial[Q_{hh}(p)]}{\partial g}\right|_{g=0}$ can be replaced by:

$$\left.\frac{\partial[Q_{hh}(p)]}{\partial g}\right|_{g=0} = \left.\frac{\partial[Q_{hh}(p)]}{\partial(ik)}\right|_{g=0} = \frac{dQ_{hh}(ik)}{d(ik)} = [Q'_{hh}(ik)] \tag{20}$$

Substituting (19) into (20) yields the approximated p-domain solution of $[Q_{hh}(p)]$ in terms of k for small g:

$$[Q_{hh}(p)] \approx [Q_{hh}(ik)] + g[Q'_{hh}(ik)] \tag{21}$$

Substituting (21) into 12 yields the *g-method* equation as follows:

$$\left[\frac{V^2}{L^2}[M_{hh}]p^2 + [K_{hh}] - q_\infty[Q'_{hh}(ik)]g - q_\infty[Q_{hh}(ik)]\right]\{q\} = \{0\} \tag{22}$$

B. Flutter Certification Procedures

Aircrafts are flown to their maximum speeds to show that they are structurally safe at those speeds. After the investigation of the flutter phenomena, flutter tests became an important part of the design and modification of the air vehicles. Fig. 1 emphasizes the verification and validation steps for an aeroelastic aircraft models

As it can be seen in Fig. 1, flutter certification is a very complicated task. Every step of this certification procedure is required a large amount of work power, time and money. Flutter certification procedure can be summarized in five steps sequentially as follows,

1. Determination of the test configurations
2. Ground vibration testing (GVT)
3. FE-model updating
4. Aeroelastic flutter analysis
5. Flight flutter tests

1. Test Configurations

Passenger planes are designed and they are used. Therefore, their flutter certification is done once. However, fighter aircrafts are designed and they are capable of carrying different type and number of external stores. Those external stores are not used arbitrarily. They are used in a concept of operation for the aircraft. The concept of operation defines the types and number of munitions that the combat aircraft carries and the location of the munitions on the aircrcraft stores. All different external store configurations have their own structural and aerodynamic identity. For each of those configurations it is necessary to re-arrange the flutter certification procedure. Determination of the test configurations is very critical since it is the starting point of the flutter test procedure.

2. Ground Vibration Testing (GVT)

After the test configurations are determined, GVT for each configurations is conducted. GVT is necessary to validate and update the mathematical model of the aircraft by using experimentally determined low-frequency modes of the whole aircraft structure. This mathematical model of the aircraft is used in flutter analysis for reliable flutter estimations.

3. FE-model updating

The results of the GVT are used for FE-model updating. Validated FE-model is used to predict the critical flutter speeds of the aircraft, then it is necessary to progress carefully in model updating stage. Due to this reason model updating procedure takes up to several weeks. Another reason for model updating to be done as accurate as possible is that the usage of the updated FE-model for the flutter calculations enables to cover future modifications on the aircraft without any additional GVT. FE-model can be updated for smaller modifications on the structure and it can be used for the flutter calculations of the modified aircraft.

4. Flutter Analysis

Updated FE-model is used in aeroelastic analysis in order to obtain an information about the flutter behavior of the aircraft. Computer programs specialized for the flutter analysis, e.g. ZAERO©, or some finite element analysis programs such as NASTRAN© can be used for flutter analysis. These results determines the safety limit for the flight flutter tests.

5. Flight Flutter Test

Flutter flight tests are the final step of the flutter certification procedure of hte aircraft. As a result of the aeroelastic analyses, most critical configurations in terms of flutter are determined. Flight flutter tests are conducted for those critical configurations. As a result of flight flutter test, flight envelope of the aircraft after the modification is determined. Structural excitation during the flutter test is necessary to detect the impending aeroelastic instabilities. For aircrafts up to 60 Hz excitation is necessary to excite selected vibration modes. Lower excitation results in lower aerodynamic damping values than the actual damping levels and a large scatter in damping values from the response data. Excitation system should be light enough not to change the modal characteristics of the aircraft. The most effective way to obtain the desired excitation is to use inertia shakers. Aerodynamic force is also a simple way to obtain the excitation force. In that type of excitation, aerodynamic vanes have a small airfoil is mounted at the tip of the wing or stabilizer. Atmospheric turbulence is also used for the excitation during the flight flutter tests. [22] It is necessary to have an excitation point far away from a nodal line [23]. Response of the aircraft to an excitation should also be recorded. Then, instrumentation is another important factor in flight flutter testing. Accelerometer is used in order to obtain the response of the aircraft during the flight flutter test. The location and the number of measurement points should be chosen carefully in order to get good enough data from the measurements. Pulse code modulation (PCM) or digital telemetry is used in order to transfer the measured responses from the aircraft to the ground station. [24] Typical characteristics of a good measurement point is that data obtained from that point should reflect the mode shapes of the aircraft. [24] After the response data obtained, it is necessary to analyze it for flutter occurence. Some of the methods used for flutter prediction are extrapolating damping trends, discrete time autoregressive moving average model and the fluttero-meter.

Fig. 2 Experimental setup for PC plate structure in Wind tunnel, Courtesy of Ankara Rüzgar Tüneli (ART)

II. Experimental Study

Within the scope of this study, plate like structure is analyzed and tested for flutter. In the wind tunnel tests, the rectangular polycarbonate (PC) plate with the dimension of 5x125x1000 mm is used. In Fig. 1 the experimental setups are given for polycarbonate (PC), rectangular plate structure. The PC-Plate is placed paralel to the flow (Fig. 2). ART is a subsonic wind tunnel which has a maximum speed of 90 m/s. Firstly, wind tunnel test of the rectangular plate is conducted. Plate is fixed to the floor of the wind tunnel with a fixture as shown in Fig. 2. Strain gauges are used to obtain strain data and they are positioned at the root of the plate.

III. Numerical Modelling and Analysis

Material properties of PC obtained from literature and used in FE model of the plate structures are given in Table II.

TABLE II
MATERIAL PROPERTIES OF PC USED IN FE MODEL

Property	Value
Elastic Modulus [MPa]	3.5
Poisson ratio	0.35
Density	1.2

Modal analyses of the plate structures have been carried out by MSC. NASTRAN©. FE models were constructed for 1/10 scaled cantilever beam plate. Fixed boundary condition and modal parameters of the FE model that correlate best with modal test data were used in flutter analyses.

A. Aeroelastic Model

Aeroelastic model of the rectangular plate obtained with ZAERO© is shown in Fig. 3. Half of the system is modeled in order to simplify the model and shorten the analysis time. The body is modeled as a cylinder and the tip of the body is sharpened in order not to affect the flow around the rectangular plate during flutter analysis.

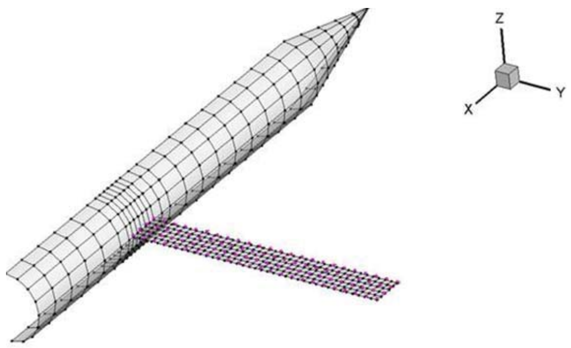

Fig. 3 Aeroelastic model of the rectangular plate

Aeroelastic model of the rectangular plate has 40 elements in spanwise direction and 5 elements in chordwise direction. The body in the aeroelastic model has 125 mm diameter in its cylindrical section.

IV. Results and Discussion

Experimental and numerical results and comparison of them are given under this topic.

A. Wind Tunnel Test Results

The strain data obtained from the strain gauges during the wind tunnel test is given in Fig. 5. As it can be seen in strain-time data, during the flutter occurrence strain values change in an uncontrolled manner. After flutter observation in test, the wind tunnel is stopped and the strain data goes back to its normal progress.

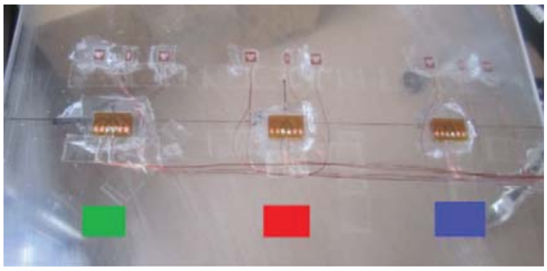

Fig. 4 Strain gage placement for the rectangular plate test

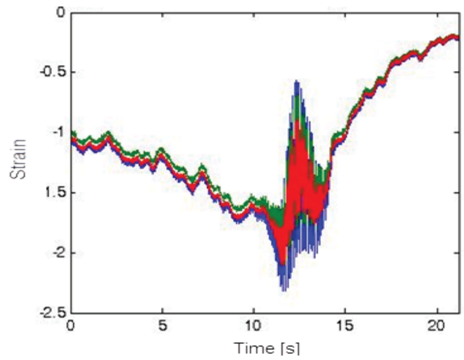

Fig. 5 Strain-time graph of the rectangular plate test item during the wind tunnel test

Three different (green, red and blue) strain gauges are attached to same horizontal line (Fig. 4). During the flutter occurence the response of the green, red and blue gauges are given in the same graph. Wind tunnel speed was 24.89 m/s when the flutter is observed at the rectangular plate as indicated in Table III.

TABLE III
WIND TUNNEL FLUTTER TEST RESULTS FOR THE RECTANGULAR PLATE

Test #	Test Configuration	Flutter Speed [m/s]
1	Rectangular plate	24.89

B. FEM Analysis Results

Modal analyses of the plate structures have been carried out by MSC. NASTRAN©. It is seen in Fig. 6 that, first four modes are first bending at 1.08 Hz, second bending at 6.75 Hz, first torsion at 14.89 Hz and third bending at 18.90 Hz in sequence for the rectangular plate. First 10 natural frequencies are also given in Table IV.

TABLE IV
FIRST 10 NATURAL FREQUENCIES OF THE RECTANGULAR PLATE

Mode No	1	2	3	4	5	6	7	8	9
Freq. [Hz]	1.1	6.8	14.9	18.9	26.5	37	44.9	61.4	91.9

Firs four mode shape are shown in Figs. 6 (a)-(d).

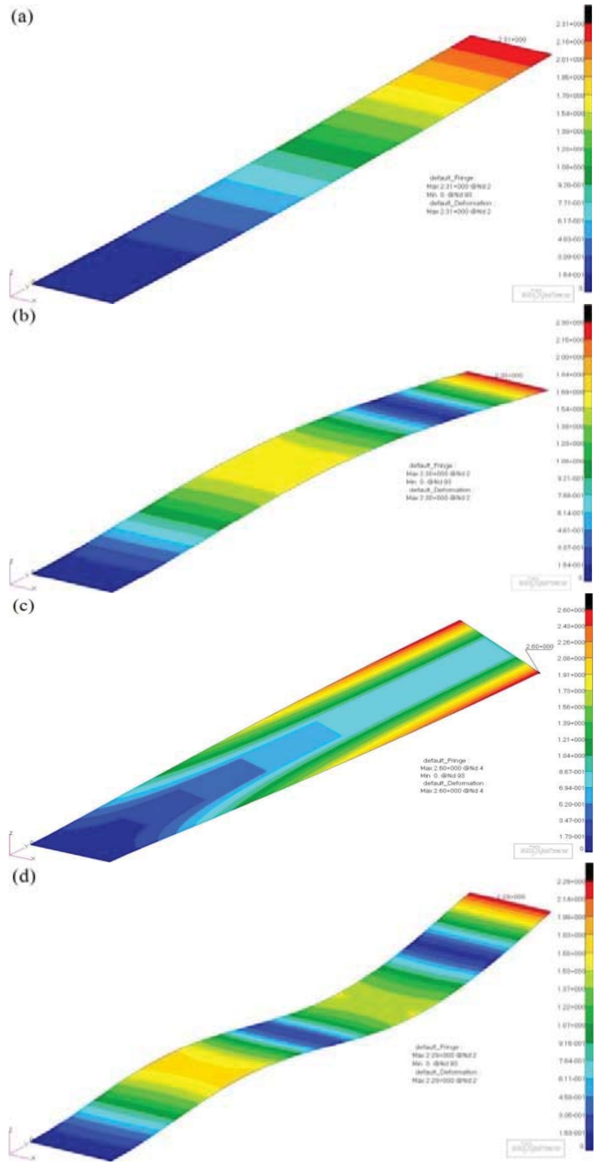

Fig. 6 First four (a:Mode1, b:Mode 2, c: Mode 3, d:Mode 4) modes of the rectangular plate

C. Flutter Analysis Results

Results of the flutter analysis for the rectangular plate indicates a flutter speed between 22.5 m/s and 23.9 m/s and a flutter frequency between 10.4 Hz and 9.7 Hz for the assumed structural damping between 0% and 4% at the third mode as shown in Table V. Fig. 7 shows the damping-speed graph (*V-g* plot). In *V-g* plot when damping of one of the modes goes from negative side to positive side it is said that speed is the flutter speed for the structure. Table V indicates flutter onset at 22.5 m/s for this case. It is also obtained that third mode causes the flutter for the rectangular plate.

1. Comparison of k–Method and g-Method Results

When the results for the rectangular plate are investigated it is seen that both of the results are in good agreement with the test result. But *k-method* solution result is better than the *g-method*. Since there is not much non-linearity in test setup for this case the results are close to each other. The general comparison between the test result and the flutter analysis is given in Fig. 8.

TABLE V
FLUTTER SPEED ANALYSIS RESULTS OF THE RECTANGULAR PLATE

Structural Damping [%]	0	0.5	1	1.5	2	2.5	3	3.5	4
Speed [m/s] (*g-method*)	22.5	22.6	22.8	23.0	23.2	23.3	23.5	23.7	23.9
Speed [m/s] (*k-method*)	23.8	24.0	24.2	24.4	24.6	24.8	25.0	25.2	25.4

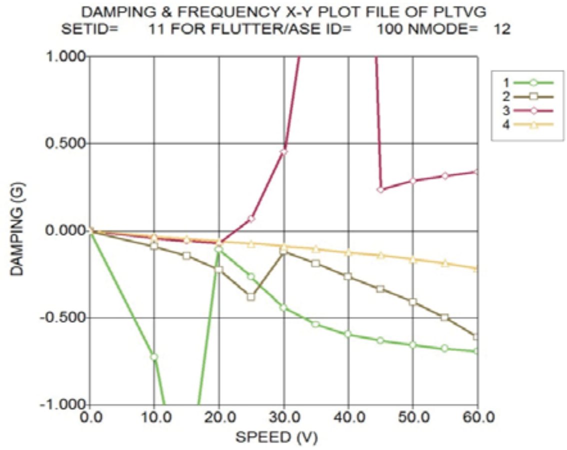

Fig. 7 *V-g* plot of the rectangular plate

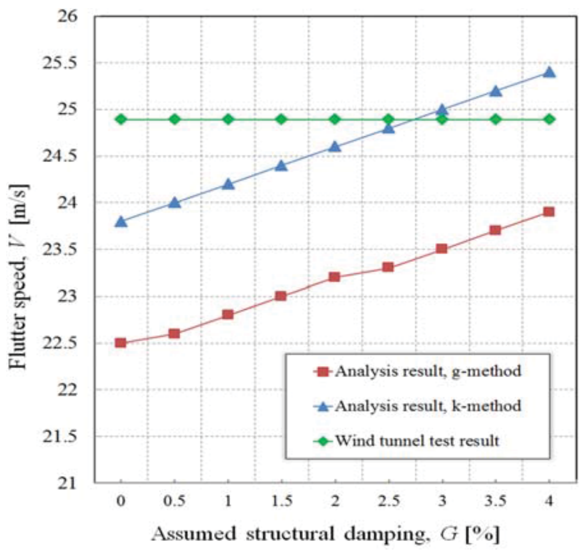

Fig. 8 Flutter speed comparison between test and analysisi result vs assumed structural damping

V. CONCLUSION

Tables VI and VII show the comparison of the *g-method* and *k-method* flutter analysis results with the wind tunnel flutter test results. It is obvious that flutter speed estimation of ZAERO© for this case is in great agreement with the wind tunnel test results for both solution methods. ZAERO© estimates the lower flutter speed than the wind tunnel test results. Therefore, it can be said that analysis results are conservative. It is also concluded that results for the *k-method* solution is better when compared to *g-method*. Flutter frequency estimation is better in *k-method* solution when compared to the *g-method*. Some reasons for so exact flutter speed estimation can be given as the system is highly linear and flutter is easily observed for this large plate in wind tunnel test.

TABLE VI
COMPARISON OF WIND TUNNEL FLUTTER TEST WITH FLUTTER ANALYSIS FOR THE RECTANGULAR PLATE FOR *G-METHOD*

ZAERO (*g-method*)			TEST		Speed Change [m/s]	Freq. Change [Hz]
Assumed Structural Damping [%]	Flutter Speed [m/s]	Flutter Freq. [Hz]	Flutter Speed [m/s]	Flutter Freq. [Hz]		
0	22.5	10.4	24.9	8.8	9.6	18.2
0.5	22.6	10.3	24.9	8.8	9.2	17.0
1.0	22.8	10.2	24.9	8.8	8.4	15.9
1.5	23.0	10.2	24.9	8.8	7.6	15.9
2.0	23.2	10.1	24.9	8.8	6.8	14.7
2.5	23.3	10.0	24.9	8.8	6.4	13.6
3.0	23.5	9.9	24.9	8.8	5.6	12.5
3.5	23.7	9.8	24.9	8.8	4.8	11.4
4.0	23.9	9.7	24.9	8.8	4.0	10.2

TABLE VII
COMPARISON OF WIND TUNNEL FLUTTER TEST WITH FLUTTER ANALYSIS FOR THE RECTANGULAR PLATE FOR *K-METHOD*

Zaero (*k-method*)			Test		Speed Change [m/s]	Freq. Change [Hz]
Assumed Structural Damping [%]	Flutter Speed [m/s]	Flutter Freq. [Hz]	Flutter Speed [m/s]	Flutter Freq. [Hz]		
0	23.8	9.7	24.9	8.8	4.4	10.2
0.5	24.0	9.6	24.9	8.8	3.6	9.1
1.0	24.2	9.5	24.9	8.8	2.8	8.0
1.5	24.4	9.4	24.9	8.8	2.0	6.8
2.0	24.6	9.3	24.9	8.8	1.2	5.7
2.5	24.8	9.2	24.9	8.8	0.4	4.5
3.0	25.0	9.1	24.9	8.8	0.4	3.4
3.5	25.2	9.0	24.9	8.8	1.2	2.3
4.0	25.4	8.9	24.9	8.8	2.0	1.1

As a result of this study, it is concluded that it is sufficient to use modal analysis results in flutter analysis for simple structures. *k-method* solution of ZAERO© is found to be more accurate than the *g-method* solution. However, *g-method* results are on the conservative side which is very critical in flutter tests since flutter tests are conducted for critical conditions of the fighter aircrafts. Then, it is better to use *g-method* results for the flutter analysis.

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