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A Numerical Description of a Fibre Reinforced Concrete Using a Genetic Algorithm

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***Abstract*—**This work reports about an approach for an automatic adaptation of concrete formulations based on genetic algorithms (GA) to optimize a wide range of different fit-functions. In order to achieve the goal, a method was developed which provides a numerical description of a fibre reinforced concrete (FRC) mixture regarding the production technology and the property spectrum of the concrete. In a first step, the FRC mixture with seven fixed components was characterized by varying amounts of the components. For that purpose, ten concrete mixtures were prepared and tested. The testing procedure comprised flow spread, compressive and bending tensile strength. The analysis and approximation of the determined data was carried out by GAs. The aim was to obtain a closed mathematical expression which best describes the given seven-point cloud of FRC by applying a Gene Expression Programming with Free Coefficients (GEP-FC) strategy. The seven-parametric FRC-mixtures model which is generated according to this method correlated well with the measured data. The developed procedure can be used for concrete mixtures finding closed mathematical expressions, which are based on the measured data.

***Keywords*—**Concrete design, fibre reinforced concrete, genetic algorithms, GEP-FC

I. Introduction

THE production of free-formed thin-walled concrete elements requires, depending on the curvature state, a targeted adjustment of the processing properties of the fresh concrete, such as, the dynamic viscosity and the time-dependent flow behavior [1], [2]. The calculation of such application-specific concrete formulations is coupled with a high experimental effort in order to meet the mechanical and rheological properties as well as requirements regarding to formwork, technology and quality.

Due to the large number of parameters and different target variables in the application-specific recipe adaptations, multicriteria optimization is essential [3]. Particular advantages are provided by optimization algorithms based on artificial neural networks as methods of nonparametric regression [4], [5]. By the aid of such algorithms, masses can be reduced, and the stiffness or strength of such structures will be increased. These methods have been already applied in the construction industry and for the development of mineral products [6]. For example, Hola and Schabowicz investigated the use of neural networks for the assessment of concrete strength by means of various non-destructive values [7]. Freitag et al. predicted the long-term deformation of textile reinforced concrete with recurrent neural networks for fuzzy sizes [8]. An analysis procedure based on artificial neural networks is also used in combination with stochastic training tests for the analysis and monitoring of the life of tunnel structures [9]. In addition to the use of optimization techniques for the design of structures, methods of artificial intelligence, in particular neural networks are used for the determination and classification of hard-to-describe material properties. These neural networks offer a way to map complicated material phenomena as a function of any input data by measured training results. In contrast to neural networks for which the mathematical model in form of a network is presented implicitly only, in this work an approach is introduced, which enables the possibility to obtain explicit mathematical expressions for an automatic adaptation of concrete formulations.

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II. Materials and Methods

A. Structure of Algorithm

In the original algorithm, GEP [10] is a combination and further development of GA [11], [12] and Genetic Programming (GP) [13]. Both are population-based optimization heuristics, which according to the principle of evolution (selection, mutation, and recombination) iteratively produce new and better-adapted individuals. These individuals will be compared with each other by assigning a quality value, which is commonly referred as fitness.

B. Coding of the Individuals

The main difference between GA and GP is the encoding of the individuals. While GP implements graphs (Parse Trees) with variable length in the GA, the representations appear as vectors with constant length. The canonical GEP algorithm also uses a tree structure (Expression Trees, cf. Fig. 1 (a)) for the representation of the mathematical expressions. This only occurs in a downstream translation step. The basic representation of the different Expression Trees and so encoded mathematical expressions is in the form of an initial vector representation (Chromosome, cf. Table I). These Chromosomes of elementary mathematical functions and argument symbols are clearly, sequentially transferred into concrete expressions.

The vectorial representation increases the expressive power on the one hand, because constant vector lengths enable the possibility of coding mathematical expressions of varying complexity. On the other hand, genetic operators can be applied to vectors in a more flexible manner. To ensure a unique transformation of the vector representation to a valid mathematical expression, certain conditions for the construction of a Chromosome must be considered.

- To further increase the flexibility, multiple sub expressions are connected to form an overall expression about the defined Linking function (e.g. add). These mathematical sub expressions are organized within the Chromosome in independent sections (Genes).
- Different Genes must have the same basic structure, when information is exchanged.
- The vector of a Gene-section is divided into a Head with h elements, which may contain function symbols and arguments also called Terminals as well as a Tail, which only contains Terminals.
- The required number t of Terminal symbols in the Tail range depends on the length h of a given Head sequence and the maximum number of function arguments n passed to it (mostly two):

$$t = h\cdot(n-1) + 1 \quad (1)$$

As an example, the basic structure of a multigenic-Chromosome is shown in Table I. It consists of three sub expressions (Genes) of the same structure (h = 3, t = 4) which are linked via a Linking function (add: addition) to a resulting overall expression see (3).

TABLE I
BASIC STRUCTURE OF A MULTIGENIC-CHROMOSOME [14]

1. CHROMOSOME ↪ LINKING: ADD						
1. Gene						
add	cos	$\sqrt{\ }$	x_1	x_2	x_2	x_1
2. Gene						
add	sin	x_2	x_1	x_1	x_1	x_2
3. Gene						
x_2	mul	add	x_1	x_2	x_2	x_2

C. GEP-FC

The aim is to obtain a closed mathematical expression which best describes a given point cloud (Target environment) by applying a genetic optimization. However, to obtain the most compact mathematical expressions, the basic GEP-algorithm will be expanded and supplemented by an additional inner optimization step.

1) Decoding of Individuals and Adaptation of the Free Coefficients

For this purpose, the function symbols of the Head sequence are not transferred in the form of elementary functions but in a general multi-dimensional family of functions. These expressions contain free coefficients w_i (GEP-FC), compare, e.g. (2).

$$\underbrace{\sin(x)}_{GEP} \rightarrow \underbrace{w_0\, sin\,(w_0\, x\, +\, w_0)\, +w_0}_{GEP-FC} \quad (2)$$

Each of free coefficients are adjusted in an inner optimization according to the Target environment, and this obtained function is associated to a fitness. These two stages allow, in contrast to the canonical GEP algorithm (2), to create a more compact mathematical expression. In the first step, the appropriate mathematical structure is built and in the second step the free coefficients are assigned with appropriate values. In Fig. 1 (a), the transfer of the multigenic- Chromosome from Table II to the Expression Tree with numbered free coefficients is shown, where the argument symbols (placeholders) #1 and #2 are successively replaced with Terminals or other mathematical expressions.

If the individual Sub Expression Trees shown in Fig. 1 (a) (according to vector Table I) are transferred to mathematical expressions and linked by addition (Linking function), the following mathematical function applies:

$$f_i(\vec{w}, x_1, x_2) = \underbrace{w_0\{w_3\cos(w_4x_1 + w_5\} + w_1\{w_7\sqrt{w_8x_2 + w_9} + w_{10}\} + w_2}_{1.Gene} + \underbrace{w_{11}\{w_{14}\sin(w_{15}x_2 + w_{16} + w_{17}\} + w_{12}x_2}_{2.Gene} + \underbrace{x_2}_{3.Gene} \quad (3)$$

This total mathematical expression contains k = 18 free coefficient $\vec{w}$ which are assigned with concrete values in a further optimization step. In order to reduce the number of possible solutions, a simplification of the term as an intermediate step will be carried out before the internal optimization takes place. Products and sums of coefficients are each summarized ($w_1w_2\ldots w_n \rightarrow w_1$, $w_1+w_2\ldots+w_n \rightarrow w_1$) by one variable. A complete reduction to unique solutions is not feasible due to the variety of possible functions and can be done only in a post-processing step (cf. (11)).

The adaptation of the free coefficients was performed by robust nonlinear optimization algorithm, NELDER-MEAD.

2) Optimization of the Mathematical Structure

Fig. 1 illustrates schematically the entire process of the two-stage optimization process. The termination is reached by a given number of generations or a function exactly approximates all test points. For an assessment of the fitness (fit_i) for the outer optimization loop as suggested by [10], the cumulative relative error (E_i) with a fitness between zero (no match) and 1000 (perfect match) according to (4) is introduced.

$$\min_{w_0 w_1 \ldots w_{k-1} \in \mathbb{R}} \left\{ E_i(\overrightarrow{w}) = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{f_i(\overrightarrow{w}, \vec{x}_j) - T_j}{T_j}\right)^2 \right\}$$

as well as

$$fit_i = \frac{1000}{1 + E_i(\vec{w}_{opt})} \quad (4)$$

To exclude invalid mathematical expressions in the domain such as division by zero, $\sqrt{\mathbb{R}^-}$ or ln $\sqrt{(\mathbb{R}^-)}$, for these proposed solutions a fitness of $fit_i = 0$ will be assigned immediately. Furthermore, individuals are selected according to their fitness for the outer optimization loop similar to a GA. The selection probability is proportional increasing to the fitness, ''Roulette Wheel selection''. To the genetic representation (Chromosome) of such selected individuals, different genetic operators will be applied with a certain probability (rate). This forms a new mathematical expression for the next iteration (generation) of the outer optimization loop. The various operators can be classified according to:

Mutation: changing a function symbol (Head) or Terminal (Tail) at a location of a Gene, wherein the structure of the Gene is to preserve.

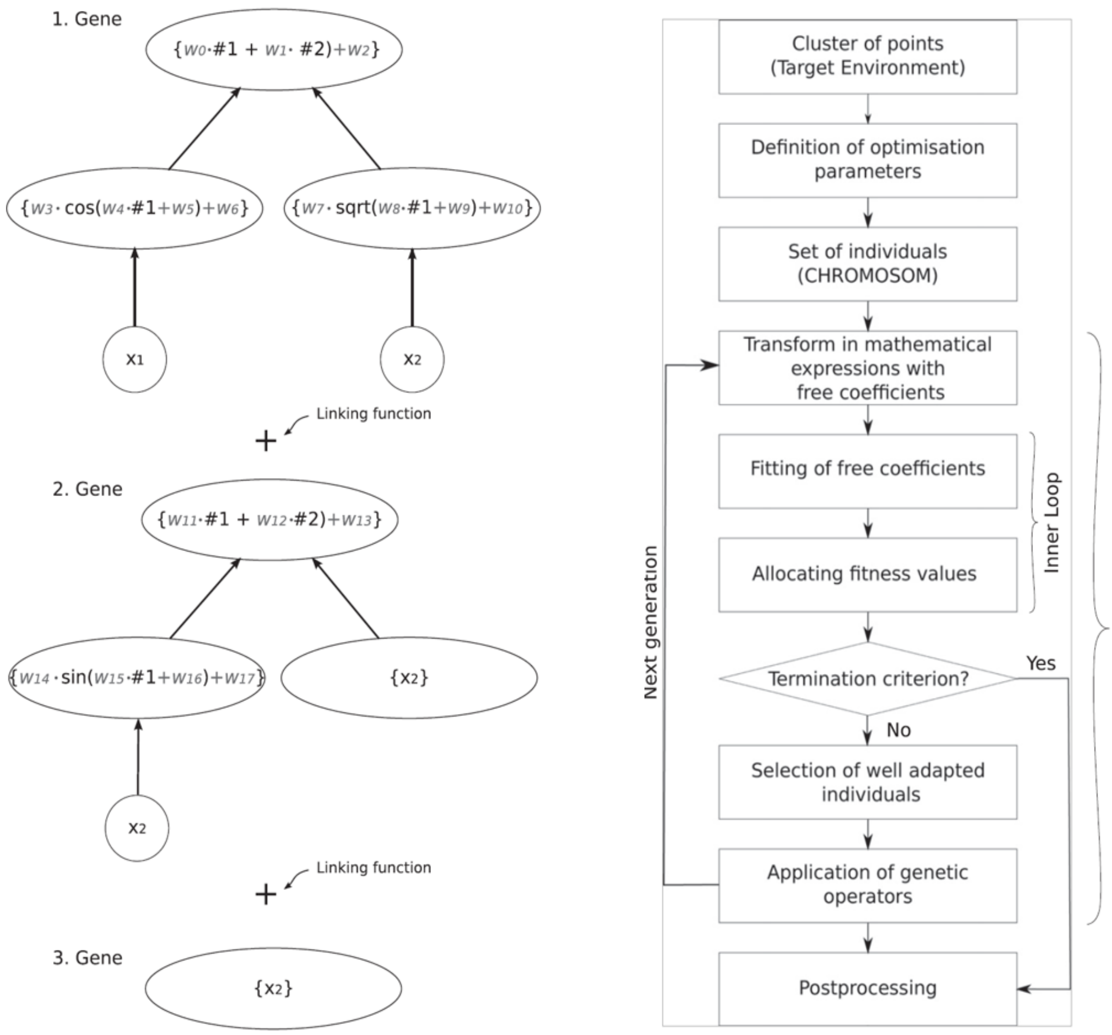
Transposition: sequences within a Gene copied to other locations within the Chromosome. The selected sequences are copied to the first place within a chosen Gene (Root Insert Sequence, RIS) or to the second place (Insert Sequence, IS). The transposition in multigenic representations can also be done in a higher level by copying of entire Genes (Gene Transposition).

Inversion: a selected sequence is randomly rearranged.

Crossover: genetic regions are exchanged between two Chromosomes. Begin to exchange from one position to the end (Single Point Crossover) or between two positions, (Two Point Crossover). The exchange of entire Genes in multigenic representations is also implemented (Gene Crossover).

The operators were implemented referring to [10], where the selection of appropriate locations, sequences as well as Genes is chosen randomly. Studies with different test functions have shown that, particularly in populations with a low number of individuals, the mutation operator can significantly contribute to the acceleration of convergence, caused by creation of new genetic material in populations. For this reason, an additional operator, the so-called Range Mutation, was implemented. Here, a contiguous sequence mutates simultaneously. Thereby, the diversification of the population is successively increased.

Fig. 1 (b) illustrates schematically the entire process of the two-stage GEP-FC optimization process. The termination is reached by a given number of generations or a function exactly approximates all test points.



(a) Expression Tree (b) Schematic flow of an optimization run

Fig. 1 GEP-FC-optimization [14]

D. Composition of FRC

Table II illustrates the qualitative and quantitative compositions of a typical FRC for façade applications. In order to test the novel approach for an automatic adaptation of concrete formulations based on GEP-FC algorithm, the compositions of the FRC was varied in one dimension. For this purpose, the water-binder (w/b ≡ x) ratio was varied, whereby the aim was to obtain only homogeneous and non-segregated fresh concrete. According to that, the w/b ratio was varied from 0.28 to 0.42. The fine concrete contained apart from white Portland Cement type 52.5 R (according to EN 206) an amorphous aluminosilicate as puzzolanic binder (Table II). Dolomite sand with a grain size of 0.1-1.0 mm was used as aggregate, and dolomite powder with an average grain size of 70 µm was used as filler. The alkali-resistant (AR) short glass fibres (16 M.-% ZrO_2) were 12 mm long and had a length mass of 45 tex. The high-performance superplasticizers had a polycarboxylate ether (PCE) content of 30 wt.%. The volumetric compensation (V = 1.00 m³) required due to the different water contents was performed over the dolomite sand.

TABLE II
COMPOSITION OF THE FRC MIXES IN KG/M³

	1 2 3 4 5 6 7 8 9 10
CEM I 52.5 R (EN 206)	500
Amorphous aluminosilicate	55
sand 0.1/1.0	1350-1120
filler (x_{50}= 74 µm)	300
Water	170-252
AR-glass fibres	5
superplasticizers	11
w/b	0.28-0.42

The fine-grained concrete was mixed with the intensive mixer Eirich R05T. The mixing parameters are shown in Table III. The mixing time was 5 min in total. The fresh concrete was tested according to DIN EN 12350.

TABLE III
MIXING PARAMETERS FOR THE PRODUCTION OF FINE CONCRETE

	COMPONENT	MIXING PRINCIPLE	MIXING POWER IN %	MIXING TIME IN S
1.	binders + aggregates	counter rotation	15	60
2.	water + super plasticizer	co-rotation	50	120
3.	AR-Glass fibres	co-rotation	60	60

E. Test Specimens and Test Setup Fibre FRC

The flow test of the fresh FRC was measured according to DIN EN 12350-5. The samples for the tests to be performed on the hardened concrete were filled into corresponding molds without vibrations and were stored dry, according to DIN EN 12390-2. The 3-point bending tensile strength (Fig. 2 (a)) was determined by means of the ToniNorm (company Toni Technik) with samples which measured 225 x 50 x 15 mm³.The span width set was 200 mm and the constant load speed was 100 N/s. The compressive strength was determined by means of the following DIN EN 12390-3, with cubes having an edge length of 150 mm (Fig. 2 (a)). The measured data points for the calculations were based on arithmetic averages, which were calculated from three measured values.

(a) Bending tensile strength (b) Compressive strength

Fig. 2 Determination of the FRC-strengths

III. RESULTS AND DISCUSSION

A. Properties of the Fresh and Hardened Concrete Mixes

TABLE IV
PROPERTIES OF FRESH AND HARDENED FRC MIXES

MIX	FLOW SPREAD IN CM	COMPRESSIVE STR. IN MPA	BENDING TENSILE STR. IN MPA
1	0	55.4	4.8
2	0	57.2	6.7
3	0	59.2	6.9
4	0	62.7	6.1
5	33	79.2	10.7
6	55	75.6	10.1
7	58	71.5	10.8
8	65	65.9	8.6
9	70	67.8	8.7
10	70	62.5	9.4

(a) Flow spread of mix 1 (b) Flow spread of mix 10

Fig. 3 Exemplary results of the flow tests

Table IV shows the characteristics of fresh concrete and hardened FRC mixes after 28 days. The fresh concrete mixes had a spreading flow between 0 and 70 cm depending on the w/b-ratio. Thus, the concrete mixes embraced all consistency classes of DIN EN 12350-5, starting from very stiff (mixture 1-4) through to self-compacting (mixture 9 and 10), see Table IV and Fig. 3. An increasing of the w/b-ratio results in an increasing of the flowability of the FRC, which corresponds to the state of the art. The compressive and the 3-point bending tensile strength of the FRC mixes ranged from 55.4 to 79.2 and 4.8 to 10.8 MPa, respectively. The comparatively low strengths of the mixtures 1 to 4 (cf. Table IV) resulted from the very stiff consistencies and the vibration-free filling of the molds.

B. Solution of the GEP-FC

To determine a quantity equation for the measured properties of the FRC (Table IV), the parameters in Table V for the GEP-FC algorithm are applied. The solution should be as compact as possible. Therefore, the allowable complexity is reduced to a sub expression (Gene) with a maximum nesting base functions of 3 (Head length *h*).

TABLE V
OPTIMIZATION PARAMETERS FOR THE NUMERICAL DESCRIPTION OF THE FRC

NUMBER OF RUNS	30
Max. number of generations	50
Population size	30
Number of fitness cases	10
Function set, two arguments	+, -, ·, /
Function set, one argument	sin; cos; tan; log; exp; $\sqrt{x}$
Terminal set	x
Head length	3
Number of genes	1
Mutation rate	0.2
Range-mutation rate	0.2
Inversion rate	0.1
Single-point crossover	0.1
Two-point crossover	0.1
Transposition IS	0.1
Transposition RIS	0.2
Fit function	(4)
Selection	Roulette wheel

After 30 independent runs of a GEP-FC algorithm, the solution was finally surrendered to the best Chromosome, as shown in Table VI.

TABLE VI
BEST CHROMOSOME AFTER 30 INDEPENDENT GEP-FC RUNS

1. GENE							
Flow spread	exp	exp	exp	x	x	x	x
Compressive strength	/	$\sqrt{x}$	sin	x	x	x	x
3-p. bending tensile strength	-	cos	sin	x	x	x	x

The optimization results of flow spread, compressive and 3-point bending tensile strength after transformation to the mathematical expression are illustrated in (5)-(7).
Flow spread

$$f(x) = w_1 + w_2\, e^{w_3 e^{w_4 e^{w_5 x}}} \quad (5)$$

Compressive strength

$$f(x) = w_1 + \frac{w_2 - w_3\sqrt{w_4 + w_5\, x}}{w_6 + \sin(w_7 + w_8\, x)} \quad (6)$$

3-point bending tensile strength

$$f(x) = w_1 + w_2\ \cos(w_3 + w_4\, x) + w_5\ \sin(w_6 + w_7\, x) \quad (7)$$

After fitting the free coefficients in (5)-(7), Figs. 4-6 show the functions of flow spread (Fig. 4), compressive strength (Fig. 5), and 3-point bending tensile strength (Fig. 6). Usually, the compressive and 3-point bending tensile strength decreases by increasing *w/b*-ratio. In this case, the mechanical strengths increased up to a *w/b*-ratio of 0.36 and then decreased again. This is due to the relative low amount of the measured data and to the very stiff consistencies of the FRC mixtures 1 to 4. Nevertheless, the parametric FRC mix model generated by the GEP-FC algorithm correlated well with the measured data (cf. Figs. 4-6.).

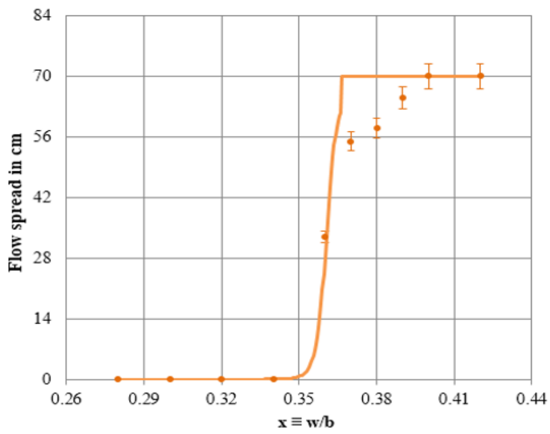


Fig. 4 Flow spread

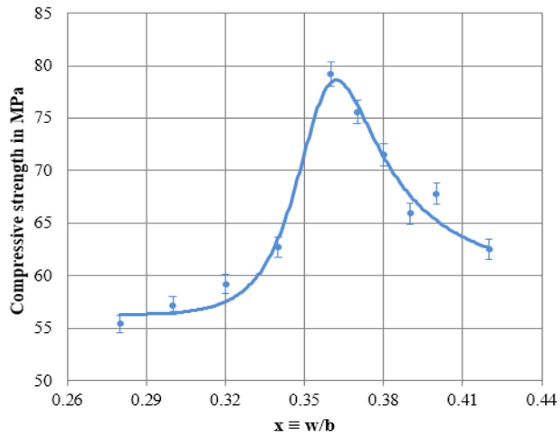


Fig. 5 Compressive strength

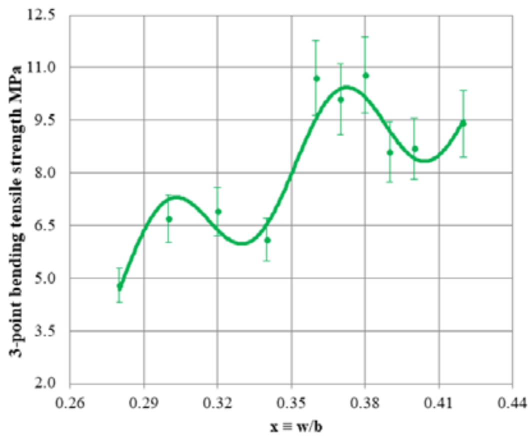


Fig. 6 Bending tensile strength

IV. Conclusion

The GEP-FC procedure can be used for finding closed mathematical expressions of FRC mixtures, which are only based on measured data. Beyond that, the significant advantage of the GEP-FC method is the ability of an automatic analysis of datasets resulting in explicit mathematical expressions, which allows a compact representation of measurement results. This has been shown by the results of the practical investigations and the subsequent mathematical calculations. The calculated mathematical expressions pointed up a good correlation with the measured data in general. This means, the function values are considered consistent with the given test points. A higher match can be achieved in principle. However, it provides only unnecessarily complex models, especially if one takes into account the measurement accuracy to be realized as well as material variations. Of course, even assuming that the measured data are completely implausible, the GEP-FC algorithm is able to generate a mathematical expression for an automatic adaptation of concrete formulations. However, this also reveals the weakness of the GEP-FC algorithm: it depends on the measurement data and the veracity of these. In the end, the algorithm allows the search for relations in the form of a closed mathematical description and thus generates a model of the material behaviour. For further analysis, only the corresponding model coefficients have to be fitted. And this results in a significant reduction of high experimental and analysis effort in order to meet the mechanical and rheological properties as well as requirements regarding to formwork, technology and quality. Moreover, it facilitates the sensitivity analysis or the integration in further computational models, which is often essential in engineering.

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