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Applying Half-Circle Fuzzy Numbers to Control System: A Preliminary Study on Development of Intelligent System on Marine Environment and Engineering

Chen-Yuan Chen, Wan-I Lee, Yi-Chaio Sui,Cheng-Wu Chen

Abstract**—**This study focuses on the development of triangular fuzzy numbers, the revising of triangular fuzzy numbers, and the constructing of a HCFN (half-circle fuzzy number) model which can be utilized to perform more plural operations. They are further transformed for trigonometric functions and polar coordinates. From half-circle fuzzy numbers we can conceive cylindrical fuzzy numbers, which work better in algebraic operations. An example of fuzzy control is given in a simulation to show the applicability of the proposed half-circle fuzzy numbers.

Keywords**—**triangular fuzzy number; half-circle fuzzy numbers; predictions; polar coordinates; Lyapunov method

I. Introduction

THE success of the application of fuzzy logic methods led to the proposal of triangular fuzzy numbers by Laarhoven and Perdrycz in 1983 [9]. Since then, triangular fuzzy numbers have often been applied in various real-world applications. Although triangular fuzzy numbers are easy and convenient to use, they still require complicated calculations in prediction [1-4]. In this study we make efforts to revise triangular fuzzy numbers to simplify these complicated operations. We call our new method half-circle fuzzy numbers. Half-circle fuzzy numbers are simpler and more convenient when applied in trigonometric functions and with polar coordinates can be used in integral or other difficult operations used for prediction. A new model based on half-circle fuzzy numbers, called cylindrical fuzzy numbers, is also developed. This new model should help make more precise predictions. We first review triangular fuzzy numbers, after which half-circle fuzzy numbers and basic polar coordinate operations are defined. The new cylindrical fuzzy number model is then applied in an example incorporating trigonometric functions and polar coordinates. Moreover, despite the many sophisticated fuzzy theories and control techniques that have been devised in the last few decades, triangular fuzzy numbers continue to be the most commonly used in the arithmetic processes (see [11-17] and the references therein).

Chen-Yuan Chen is with Department of Computer Science. National Pingtung University of Education, Pingtung 90003, Taiwan
Wan-I Lee is with Department of Marketing and Distribution Management National Kaohsiung First University of Science and Technology Kaohsiung 811, Taiwan
Yi-Chaio Sui is with Institute of Business Administration National Kaohsiung First University of Science and Technology,Kaohsiung 811, Taiwan
Cheng-Wu Chen is with Institute of Maritime Information and Technology National Kaohsiung Marine University, Taiwan.
e-mail:chengwu@mail.nkmu.edu.tw

Triangular fuzzy numbers have the advantage of being simple representations so can be easily implemented by the great majority of industrial practitioners and control designers. However, to the best of our knowledge, the issues of application based on half-circle fuzzy numbers have seldom been discussed. To fill this gap we develop a half-circle fuzzy number model based on leading polar coordinates for use with trigonometric functions. We hope that fuzzy numbers can be applied in linear algebra, Laplace equations, differential equations, statistics, and other areas.

Recently, fuzzy theory and artificial intelligence have been successfully applied to the nonlinear system such as robot manipulation, engineering application and management (see [18-51] and the references therein). The fuzzy numbers are also accompanied to solve the prediction or forecasting problems. However, the comparison of all kinds of fuzzy numbers in fuzzy control is seldom discussed. This study will propose triangular and half-circle fuzzy numbers and demonstrate the control feasibility in T-S fuzzy systems.

II. Related Work and the Novel Fuzzy Number Model

In Mendel's 1995 report, membership functions were for the most part, associated with terms that appeared in the antecedents or consequents of rules, or in phrases. The most commonly used shapes for membership functions are triangular, trapezoidal, piecewise linear or Gaussian. Until very recently, membership functions have been chosen arbitrarily, based on user experience. As a consequence, the membership functions can be quite different for different users depending upon their experience, perspective, cultural background, etc. There are three main parts included in fuzzy inference: quantification, fuzzification and defuzzification. Membership function is a part of fuzzy inference.

Triangular fuzzy numbers

Triangular fuzzy numbers as proposed by Laarhoven and Pedrycz (1983) are described below [5-6].

Definition 1. A fuzzy number M on R (=(-∞,+∞)) is a triangular fuzzy number if its membership function $\mu_M : \mathrm{R} \rightarrow [0,1]$ is equal to (1)

$$\mu_M(x) = \begin{cases} \frac{1}{m-l}x - \frac{l}{m-l}, & x \in [l, m] \\ \frac{1}{m-u}x - \frac{u}{m-u}, & x \in [m, u] \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $l \le m \le u$; l and u respectively stand for the lower and upper values in support of M; and m stands for the model value. The triangular fuzzy number, as given by equation (1), can be denoted by (l, m, u). The hypothesis of M is given as a set of elements $\{x \in R \mid l < x < u\}$. The membership function of the triangular fuzzy number is shown in Figure 1 [7-8, 10].

Operations using fuzzy numbers are illustrated below. Consider the following two triangular fuzzy numbers:

$M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$

- Addition

$$M_1 + M_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$

- Subtraction

$$M_1 - M_2 = (l_1 - l_2, m_1 - m_2, u_1 - u_2)$$

- Multiplication

$$M_1 \times M_2 = (l_1 l_2, m_1 m_2, u_1 u_2), l_1, l_2 \ge 0$$

- Division

$$M_1 \div M_2 = (l_1 / u_2, m_1 / m_2, u_1 / l_2), l_1 \ge 0, l_2 > 0$$

Half-circle fuzzy numbers

A model of half-circle fuzzy numbers developed based on triangular fuzzy numbers is described in detail below.

Definition 2. A fuzzy number H is defined on R (=(-∞,+∞)) as a half-circle fuzzy number if its membership function $\mu_H : \mathrm{R} \to [0,1]$ is equal to (2)

$$\mu_H(x) = \begin{cases} \sqrt{1-(x-h)^2}, & x \in [h-1, h+1] \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The membership function of a half-circle fuzzy number is shown in Figure 2.

Now, the operations for fuzzy numbers are presented. Consider two triangular fuzzy numbers:

$H_1 = (h_1 - 1, h_1, h_1 + 1)$ and $H_2 = (h_2 - 1, h_2, h_2 + 1)$

- Addition

$$H_1 + H_2 = (h_1 + h_2 - 2, h_1 + h_2, h_1 + h_2 + 2)$$

- Subtraction

$$H_1 - H_2 = (h_1 - h_2 - 2, h_1 - h_2, h_1 - h_2 + 2)$$

- Multiplication

$$H_1 \times H_2 = ((h_1 - 1)(h_2 - 1), h_1 h_2, (h_1 + 1)(h_2 + 1)), h_1, h_2 \ge 1$$

- Division

$$H_1 \div H_2 = ((h_1 - 1)/(h_2 - 1), h_1 / h_2, (h_1 + 1)/(h_2 + 1)), h_1, h_2 > 1$$

With polar coordinates

Polar coordinates are utilized to develop the circle function in half-circle fuzzy numbers as follows (Fig. 3):

$$f(r,\theta): r^2 + r_0^2 - 2r_0 r\cos(\theta - \theta_0) = a^2, \ a \in R,$$

where (r_0, θ_0) is the center of the circle described by polar coordinates with radius a.

We can now define half-circle fuzzy numbers using polar coordinates.

Definition 3.

$$\mu(r,\theta) = \begin{cases} r^2 + r_0^2 - 2r_0 r\cos(\theta - \theta_0) - 1, & r \in [r_0 - 1, r_0 + 1] \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

II. EXAMPLE

From the above mentioned two kinds of fuzzy numbers, triangular and half-circle fuzzy numbers, an example of T-S fuzzy control is given to compare the control effectiveness of these two. Before discussing the simulation example, a useful inequality is provided below.

Lemma 1 [13]: The equilibrium point of a closed-loop fuzzy system $\dot{x}(t) = \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t) h_l(t) \{(A_i - B_i K_l) x(t)\}$ is asymptotically stable in the large, if there exists a common positive definite matrix P such that

$$(A_i - B_i K_l)^T P + P(A_i - B_i K_l) < 0, \quad (4)$$

where $P = P^T > 0$, and $i, l = 1, 2, \cdots, r$.

The objective of this section is to choose the proposed triangular and half-circle membership functions which satisfy Eq. (4). Here we have the following:

Consider a nonlinear system described by the following equation [12]:

$$\begin{cases} \dot{x}_{11}(t) = -29\, x_{11}(t) + 1\, x_{21}(t) - 0.5\, x_{21}^2(t) \\ \dot{x}_{21}(t) = 3x_{11}(t) - 12x_{21}(t) - 0.5x_{21}^2(t) + u(t) \end{cases} \tag{5}$$

Step 1: We establish a T-S fuzzy model for the above system and the nonlinear system (5) can be approximated by the following fuzzy models:

Rule1: IF $x_1(t)$ is M_{11} THEN $\dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t)$;

Rule2: IF $x_1(t)$ is M_{21} THEN $\dot{x}_1(t) = A_2 x_1(t) + B_2 u_1(t)$, (6)

where

$x_1^T(t) = [x_{11}(t)\ \ x_{21}(t)]$, $A_1 = \begin{bmatrix} -29 & 1 \\ 3 & -12 \end{bmatrix}$, $A_2 = \begin{bmatrix} -29 & 0.5 \\ 3 & -12.5 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and the triangular and half-circle membership functions for Rule 1 and Rule 2 are plotted in Figs. 4-5.

Step 2: In order to stabilize system (5), the feedback gains and common P should satisfy Eq. (4). The optimal matrices shown below are obtained via the Matlab LMI optimization toolbox.

$K_1 = \begin{bmatrix} -11.48 & -0.37 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.51 & 0.15 \end{bmatrix}$, $P = \begin{bmatrix} 1.50 & -0.28 \\ -0.28 & 1.76 \end{bmatrix}$.

The simulation results of triangular and half-circle fuzzy numbers are illustrated in Figs. 6-7. The initial conditions are $x_1(0) = 1.7$, $x_2(0) = -2.2$.

In can be seen that system (4) is asymptotically stable; system trajectories starting from non-zero initial states asymptotically approach the origin. From Figs. 6-7, we see the control results of two kinds of fuzzy numbers are almost the same.

III. Conclusions

The reason why triangular fuzzy numbers are so often used by researchers is that they make calculation simple and easy. In this study we successfully develop a half-circle fuzzy number model and polar coordinates. These half-circle fuzzy numbers are not only simple and easy to use but also make it easy to simulate different data. Some types of problems are especially suitable for half-circle fuzzy numbers. In addition, the intersection of half-circle fuzzy numbers is larger than with triangular fuzzy numbers making them more accurate if we want to find the answer under some condition. Here, we use polar coordinates to transform half-circle fuzzy numbers from a rectangular coordinate system, introducing the triangular function to the fuzzy number function. The triangular function is easier in terms of calculation and for the simulating of dispersed data. A simulation example is utilized to demonstrate the applicability of the proposed half-circle fuzzy numbers. The results show the control effectiveness are almost the same while using different fuzzy numbers in T-S fuzzy control.

IV. Discussion

Taiwan is a maritime country. Most of each county all border with the ocean and on salt water. Many demands and pressures are placed on Taiwan's ocean problem and marine environment. Classic sectors such as transportation, shipbuilding and fishing are being joined by other uses. They include offshore gasoline, recreational fishing, aquaculture, eco-tourism and pleasure boating. The awareness of intelligent system is giving rise to current marine environment and engineering, which makes this roadmap necessary.

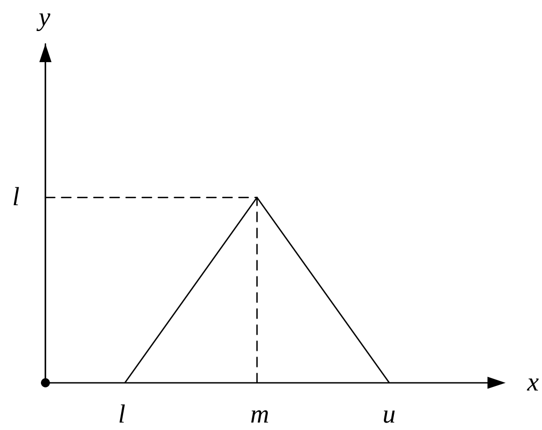

Fig. 1 Membership function of a triangular fuzzy number

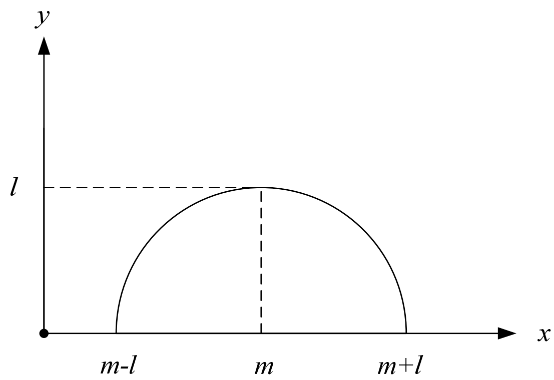

Fig. 2 Membership function of a half-circle fuzzy number

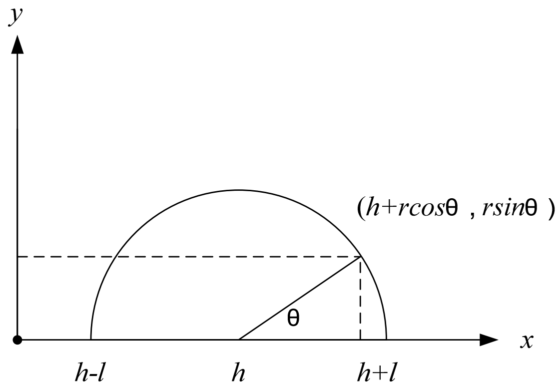

Fig . 3 Membership function of a half-circle fuzzy number using polar coordinates

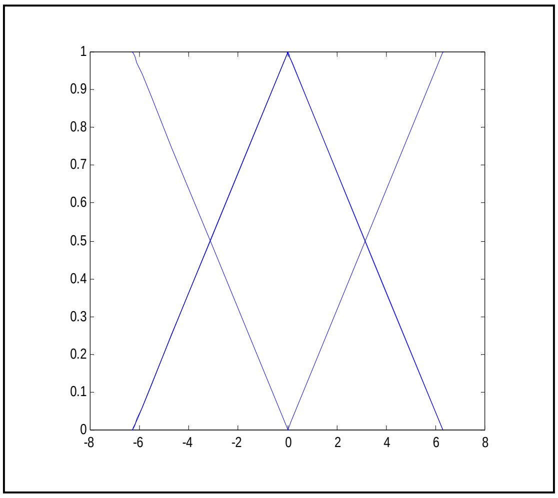

Fig. 4 The membership function of triangle.

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