A Practical Method for Load Balancing in the LV Distribution Networks
Case study: Tabriz Electrical Network

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Abstract—In this paper, a new efficient method for load balancing in low voltage distribution systems is presented. The proposed method introduces an improved Leap-frog method for optimization.

The proposed objective function includes the difference between three phase currents, as well as two other terms to provide the integer property of the variables; where the latter are the status of the connection of loads to different phases. Afterwards, a new algorithm is supplemented to undertake the integer values for the load connection status. Finally, the method is applied to different parts of Tabriz low voltage network, where the results have shown the good performance of the proposed method

Keywords—Load balancing, Improved Leap-frog Method, Optimization algorithm, Low voltage distribution systems.

I. INTRODUCTION

DEVELOPING in distribution power systems, load variety and loads sensitivity have made distribution companies to pay special attention to power quality indices and networks reliability. One of the important topics in low voltage distribution systems is loss reduction, in order to reduce the costs.

The existing low voltage distribution systems have various single, two and three phase loads. Optimum distribution of single phase and double phase loads between three phases network is one of the important factors in reduction of the difference in the amplitude of loads between the three phases and power losses consequently.

In this paper a practical algorithm for load balancing in LV distribution networks in presented which is based on applying Modified Leap Frog Method to optimization of loads' connections to different phases subject to the fact that each single phase can be connected to one of the phases and the variable parameter which indicates this connectivity should remain as an integer number through and in final stage of optimization process.

By applying the proposed algorithm, the neutral current becomes very low and power losses due to unbalancing decrease significantly.

II. MODELING

Many different objectives could be considered for optimization function. In this paper the difference between amplitude of three phase current, in least square function form, is used as the objective function(similar to[1]).
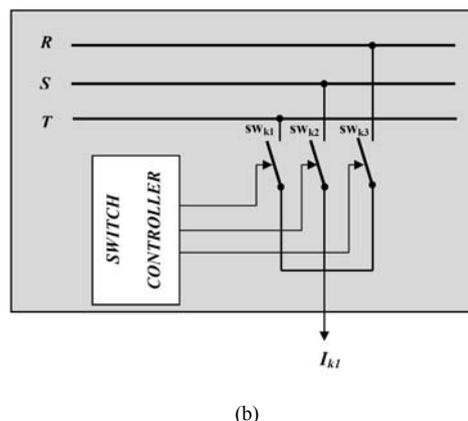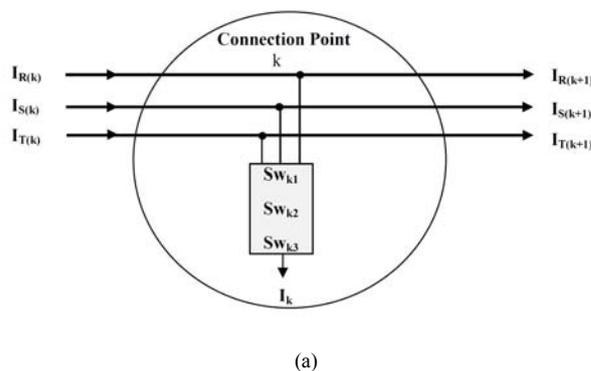

(a)

(b)

Fig. 1. (a) Distribution Feeder around point k

(b) Load Switching Configuring for a load

Considering a distribution feeder, as shown in Fig (1.a); I_{K1}, I_{K2}, I_{K3} are three loads connected to the network by means of

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virtual switches sw_{11} through sw_{33} at point k. Suppose that point k is a bus bar in an actual network; thus, several loads can connect to point k, whichever could be connected to one of the phases, depending on sw_{11} through sw_{33} situation (See Fig 1-b). The least squares objective function can be expressed as

$$J = (I_R - I_S)^2 + (I_R - I_T)^2 + (I_S - I_T)^2 \qquad (1)$$

where I_R, I_S and I_T represents the phase currents (phasors) at the feeding point. According to Fig.1, the phase currents feeding the point k can be introduced as:

$$I_{Rk} = sw_{k1}I_k + I_{R(k+1)} \qquad (2)$$

$$I_{Sk} = sw_{k2}I_k + I_{S(k+1)} \qquad (3)$$

$$I_{Tk} = sw_{k3}I_k + I_{T(k+1)} \qquad (4)$$

Therefore three phase currents, as functions of switches, are independent variant in objective function J.
According to the fact that each single phase load can be connected to one of the phases, thus for each load it can be written:

$$\sum_{i=1}^{3} sw_{ki} - 1 = 0 \qquad (5)$$

So the above relation can be rewritten, as a constraint in the optimization process as follows:

$$C_k = (\sum_{i=1}^{3} sw_{ki})^2 - 1 \qquad (6)$$

III. LEAP-FROG OPTIMIZATION ALGORITHM

There are many solvers to optimize an objective function. Gauss-Newton and Lagrange multipliers are well-known methods among them. To use them it is necessary to derive expressions for the gradient vector and Hessian matrix. It is very difficult and sometimes impossible to calculate the inverse of these matrices. Therefore, it becomes intricate to use these solvers to converge the switching matrix system.

The LFOP is different from other gradient-based methods and its advantage is independence from Jacobean and Hessian matrices. This method is based on the motion of a particle of unit mass in an n-dimensional conservative force field, where the total particles' energy consists of the kinetic and positional energy is constant [1].

IV. PROPOSED OBJECTIVE FUNCTION

Previously defined J function in (1), by considering equations (2)-(4) is considered as function of switches (sw_{ki}). Afterward, by considering constraints, the combination is used as a new penalty function [6].

$$P(sw_{ki}, \alpha) = J(sw_{ki}) + \sum_{k=1}^{n} \alpha C_k \qquad (7)$$

where,
C_k: are problem constraints defined by (6)
n: number of system loads
α: a constant positive multiplier

Because, the load balancing method using above function has not shown an acceptable integer results, in this paper an additional modifications are applied to this function. To achieve acceptable answers, i.e. solutions in the form of integer variables, in addition to formerly mentioned constraints by (6), a new constraint is necessary, i.e.

$$F_k = 100sw_{k1} + 10sw_{k2} + sw_{k3} \qquad (8)$$

$$C_k^{new} = (F_k - 100)^2 (F_k - 10)^2 (F_k - 1)^2 \qquad (9)$$

This new function maintains the integer property of the switching parameters (sw), and is calculated for each of the system loads with only changing the equation multipliers periodically in optimization process. Therefore, the new objective function could be defined as below:

$$P(sw_{ki}, \alpha) = J(sw_{ki}) + \sum_{k=1}^{n} \alpha C_k + \sum_{k=1}^{n} \beta C_k^{new} \qquad (10)$$

where, β is an additional positive factor

In the next step, the gradients of defined function on the independent variables, i.e. switches conditions, are calculated. Using the presented optimization algorithm, the next step solution is calculated based on initial switches conditions and computed gradients values. This process in continued until the switches' changes between two steps become small. In that case, it could be considered that the penalty function and consequently the load balancing function will have their best values.

However, that output optimum solution of LFOP algorithm in some cases differs slightly from optimum load value solutions based on variations in load values and network configuration. Therefore, to compensate those variances the outputs of LFOP algorithm are fed to an innovative sub-function. Finally, is illustrated in Fig. 2, the compensated results considered are final switching matrix. This combinational algorithm is referred to as Modified LFOP (MOLFOP) in this paper.

V. INNOVATIVE COMPENSATOR SUBROUTINE

As mentioned before, the output of LFOP algorithms depends on network configuration and loads value, so that sometimes they cannot be considered as an optimal solution. Therefore, using an additional sub-function for compensation of the output solution is vital (Fig.3).

The first step toward the definition of such supplementary algorithm is the definition of an index for network imbalance [1].
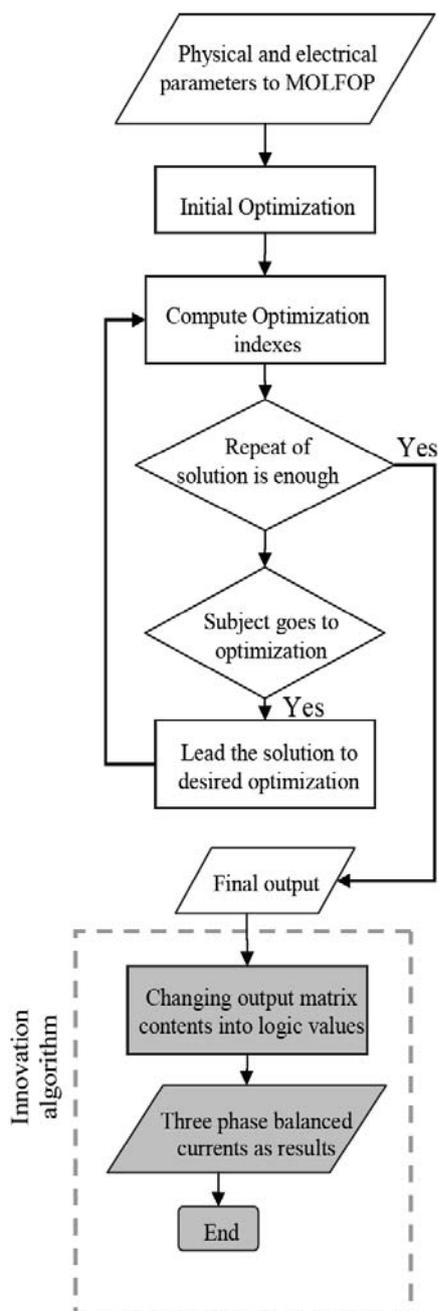
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Fig. 2. The MOLFOP Algorithm for Load Balancing

Fig. 3. The compensator Subroutine

$$\beta = \sqrt{\frac{I_M - I_m}{I_M + I_m}} \qquad (11)$$

where, I_M and I_m is the maximum current and minimum current in the three phases of the feeder under consideration, respectively. A brief description of this algorithm could be presented as below:

a) The imbalance index of switching matrix obtained from MOLFOP algorithm (β_{LFOP}) is compared with a predefined threshold value (β_{max}). In the case of lower values,

($\beta_{LFOP} < \beta_{max}$), the algorithm is terminated and the switching matrix is used as the optimum configuration.

b) Otherwise, in case of higher values for β_{LFOP}, a new parameter is defined, using the following relation:

$$\Delta I = \frac{I_M - I_m}{2} \qquad (12)$$

Now the loads to be transferred between the phase with current I_M and the phase with current I_m , named as effective load are determined as:

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$$\left\{ I_k \in I_M, \left| I_k - \Delta I_{ef} \right| < \Delta I_{ef} \right\} \qquad (13)$$

Then, the effective load with lower value is moved from maximum current phase (I_M) to minimum one (I_m).

Subsequently, the current differences index (ΔI) is recalculated and above steps is repeated until the set in (13) have not any other member.

c) In this step a new set of load buses is constructed as defined in Eq. 14, and the procedure in the pervious step is also applied on this set.

$$\left\{ I_k \in I_M, I_j \in I_m, \left| (I_k - I_j) - \Delta I_{ef} \right| < \Delta I_{ef} \right\} \qquad (14)$$

However, the procedure reaches the maximum number of iterations the procedure is terminated and the switching matrix with minimum value of β is used as the final optimum load configuration.

To compare the results with the heuristic (HE) and NN algorithms presented in [8] the algorithm is applied to the same three test systems, as shown in Table 1.(SM means switching matrix, i.e. the load status of connection to different phases)

In Table 2 the output switching matrix of the three algorithms are presented. In addition, the three phase currents of network after load balancing procedures are presented in Table 3.

TABLE I
LOAD CONNECTIONS BEFORE BALANCING

L-n	1ST Data Set		2ND Data Set		3RD Data Set	
	I_L(A)	SM	I_L(A)	SM	I_L(A)	SM
1	94.06	R	40.16	R	1.51	R
2	22.88	S	92.61	S	73.93	S
3	60.07	T	90.77	T	44.06	T
4	48.11	R	40.61	R	92.24	R
5	88.23	S	88.47	S	46.13	S
6	75.44	T	5.73	T	41.44	T
7	45.19	R	34.93	R	83.77	R
8	1.83	S	80.50	S	51.99	S
9	81.31	T	0.97	T	20.06	T
10	60.92	R	13.75	R	66.54	R
11	78.4	S	20.07	S	82.97	S
12	91.25	T	19.67	T	1.94	T
13	73.08	R	59.77	R	67.44	R
14	17.45	S	26.94	S	37.56	S
15	44.02	T	19.68	T	82.34	T
Phase current summary						
I_R(A)	321.36		189.22		311.5	
I_S(A)	208		301.23		252.58	
I_T(A)	260.84		143.58		189.84	
ΔI(A)	113.36		157.65		121.66	

TABLE 2
switching matrix and load arrangements

SM	1ST Data Set			2ND Data Set			3RD Data Set		
L-n	NN	HE	MO	NN	HE	MO	NN	HE	MO
1	R	R	S	R	R	R	R	R	S
2	S	S	R	S	R	R	S	S	T
3	R	R	R	T	S	T	T	T	R
4	T	T	T	R	T	R	R	S	S
5	R	T	T	T	T	T	R	R	R
6	R	T	R	R	R	R	S	S	R
7	S	S	S	T	T	T	T	T	S
8	T	T	R	R	S	R	R	R	R
9	S	T	T	S	S	R	T	S	R
10	R	R	S	R	R	S	S	T	R
11	T	S	R	T	S	R	S	R	T
12	S	R	S	S	S	R	T	T	S
13	S	S	T	T	R	S	R	T	T
14	S	S	R	S	T	R	R	S	T
15	R	R	R	S	T	R	R	R	S

VI. ALGORITHM IMPLEMENTATION IN A REAL NETWORK

In order to verify the practicality of the proposed algorithm, it is applied to two low voltage feeders of Tabriz Electric Distribution Company, shown in Fig 4 and 5.

Feeder loads are given in Table 4 and 5.(the loads include some three and single phase loads).

Single phase lines are as single phase loads in branching point. To measure the greatest unbalancing, currents of loads are measure in daily peak load time. In addition, it is assumed that the currents have constant value in these case studies. Each of these networks, selected from Tabriz residential regions, have intense unbalancing because of abundant building constructions.

Since the loads are house-holds, it is generally accepted that the power factors are similar and the load balancing is performed on the amplitudes of currents.

Table 6 has exhibited the three phases and neutral currents value in the beginning of feeder at a selected hour. As it is shown, the residual current has about 36 percentage of minimum phase current value (I_m).

Table 7 has shown the load and neutral currents after applying the proposed method which processes the suitability of implying the proposed method for load balancing in LV networks.

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TABLE III
CURRENT DISTRIBUTION IN DIFFERENT PHASES AFTER BALANCING

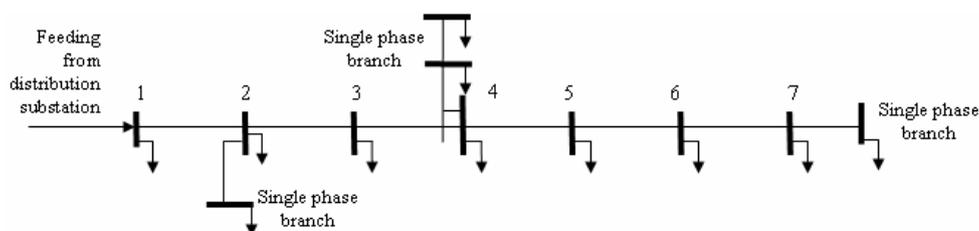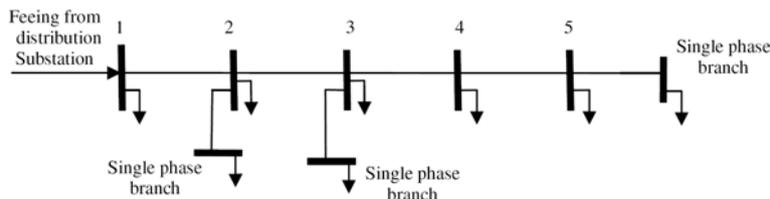|	1ST Data Set			2ND Data Set			3RD Data Set		
	NN	HE	MO	NN	HE	MO	NN	HE	MO
$I_R(A)$	270.9	290.8	300	175.5	208.3	214.1	299.6	262.3	270.2
$I_S(A)$	304.1	299.5	290.8	245.2	210.6	206.2	227.4	267.9	261.8
$I_T(A)$	307.3	291.9	291.3	213.9	215.8	214.1	266.9	263.7	261.9
$\Delta I(A)$	36.4	8.7	9.2	69.7	7.5	7.8	72.2	5.6	8.42

Fig4. Javidkia alley configuration

Fig5. Zareiy alley configuration

TABLE IV
JAVIDKIA ALLEY LOAD ARRANGEMENT

node	Single line (A)	Three phase load (1)(A)			Three phase load(2)(A)			Single phase loads (A)						
1	-	-	-	-	-	-	-	6.3	-	-	-	-	-	-
2	10.5	14.7	13.2	10	8	5.2	3.6	-	-	-	-	-	-	-
3	-	24.5	13.5	20	-	-	-	3.3	6	3	3.4	7.4	5.5	5
4	18.6	21.3	15	7.9	9	4	12.6	8	5.1	2	-	-	-	-
5	-	-	-	-	-	-	-	5.5	7.6	3	-	-	-	-
6	-	16.2	8.8	9	-	-	-	6.2	4.3	-	-	-	-	-
7	15	-	-	-	-	-	-	4.8	3.5	7	-	-	-	-

TABLE V
ZAREIY ALLEY LOADS ARRANGEMENT

node	Single line(1) (A)	Single line(2) (A)	Three phase load(A)			Single phase load (A)					
1	-	-	21.5	10.5	12	4.3	8	15.7	-	-	-
2	25.5	-	-	-	-	8.1	12	4	-	-	-
3	17	30.8	38	15	10	3.6	7	-	-	-	-
4	-	-	-	-	-	6	5	5	11.4	8.5	9.3
5	14	-	23.2	51.8	39	-	-	-	-	-	-

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TABLE VI
NETWORK LOADS BEFORE OPTIMIZATION

Network number	Iph_1 (A)	Iph_2 (A)	Iph_3 (A)	β	In (%)I_m
1	111.6	143.1	102.8	0.4	35.7
2	113.8	162.4	140	0.42	37.02

TABLE VII
NETWORK LOADS AFTER MOLFOP ALGORITHM APPLYIED

Network number	Iph1 (A)	Iph2 (A)	Iph3 (A)	β	In (%)Im
1	118.2	119.5	119.8	0.081	1.24
2	141.8	139.4	135	0.156	4.42

VII. CONCLUSION

Load balancing in low voltages distribution feeders, is a vital for loss reduction, for which an accurate and reliable algorithm is presented in this paper. Employing the proposed method results in the optimized single phase of a distribution feeder.

The proposed optimization method take the advantage of a new defined cost function and constraints as well as an innovative post-algorithm in order to present the optimized load arrangement while keeping the integer property of elementary switching matrices.

The results have shown better balancing between the currents in three phases rather than some of published methods.

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