

# Joint Optimization of Pricing and Advertisement for Seasonal Branded Products

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**Abstract**—The goal of this paper is to develop a model to integrate “pricing” and “advertisement” for short life cycle products, such as branded fashion clothing products. To achieve this goal, we apply the concept of “Dynamic Pricing”. There are two classes of advertisements, for the brand (regardless of product) and for a particular product. Advertising the brand affects the demand and price of all the products. Thus, the model considers all these products in relation with each other. We develop two different methods to integrate both types of advertisement and pricing. The first model is developed within the framework of dynamic programming. However, due to the complexity of the model, this method cannot be applicable for large size problems. Therefore, we develop another method, called hierarchical approach, which is capable of handling the real world problems. Finally, we show the accuracy of this method, both theoretically and also by simulation.

**Keywords**—Advertising, Dynamic programming, Dynamic pricing, Promotion.

## I. INTRODUCTION

SEASONAL products are a group of products with a limited life time. Fashion clothes are considered as an example. If seasonal products are not sold during a specific period of time, they cannot be sold afterwards or are sold at a low salvage price. Thus, all products must be sold during the same season. Otherwise, due to the nature of fashion industry the remaining stocks of these products will not be popular in the next season and thus it is not cost-efficient to store them for possible future sale.

In the literature, one can find a numerous number of studies regarding both pricing and seasonal products. Coulter [1] suggests that revenue management is appropriate in ‘seasonal’ retailing industry in which capacity (inventory) is not necessarily ‘perishable’ but the value of the capacity may decline significantly after the selling season. He investigates how to use discount pricing to maximize the revenue gained from selling a ‘seasonal’ product. Aviv and Pazgal [2] perform a quantitative analysis on how to apply dynamic pricing to sell fashion-like goods for ‘seasonal’ retailer. Choi [3] investigates the pre-season inventory and pricing decisions for fashion retailers. Hatwin [4] and Lippman [5] focus on issues such as pricing strategy, market share preservation, and

customer loyalty when applying revenue management techniques on grocery retail outlets, and Modarres and Bolandifar [6] consider opportunistic cancellation as a promotion strategy for seasonal products and develop an integrated model for pricing and overselling. For more information, see McGill and Van Ryzin [7] and Wen-Chyuan Chiang, et al. [8].

There has also been extensive literature on pricing and promotion. These two concepts are considered by the marketing team and have been studied widely in marketing literature [9], [10]. The focus of these studies is how promotion and price influence the demand. Little [10] proposed a marketing-mix model in which the effects of advertising, promotions, price and retail distribution on demand are independent, deterministic and addable. Dube´ et al. [9] developed a model in which the price influences the demand deterministically and the advertisement influences the demand via a Markov process. They showed that the model fits nicely with the data from the Frozen Entee´ product category. For more information see Raju[11]

Similarly, the researches, as well as practitioners, have developed new ideas, which can be found in the literature. Yu-Chung Tsao and Gwo-Ji Sheen [12] studied the problem of dynamic pricing, promotion and replenishment for a deteriorating item subject to the supplier’s trade credit and retailer’s promotional effort. They adopted a price and time dependent demand function to model the finite time horizon inventory for perishable items. Their objective was to determine the optimal retail price, the promotional effort and the replenishment quantity so that the net profit is maximized. Ju-Liang Zhang et al. [13] provided an analytical model for obtaining optimal decisions jointly on pricing, promotion and inventory control. Specifically, they studied a single item, finite horizon, periodic review model in which the demand is influenced by price and promotion, and the objective is to maximize the total profit. They considered the promotion as a binary variable. Numerical study was also provided to demonstrate the contribution of their model. In both mentioned researches, the integrated model is considered for a single product.

We assume two classes of sale promotion. The first one is promoting the sale of a particular product and the other one is to promote the brand. The goal of this paper is to determine the optimal price of each product, the suitable level and the type of advertising for each product as well as for the brand. It

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is obvious that the promotional efforts to advertise the brand will affect the demand of all products of the brand. Therefore, our proposed model includes all the products of the brand. The objective of this paper is to develop an approach to determine the optimal price, products promotion and brand promotion simultaneously, in order to maximize the total revenue. In fact, what makes our approach different from the previous ones in the literature is that we consider multiple products and two classes of advertisements within a single approach. We also develop two methods to solve the models accordingly.

The paper is organized as follow. In Section 2, the problem definition and the assumptions are introduced. In Section 3, we develop two different methods. In Section 4, to illustrate the method and its efficiency, we present some numerical examples and the computational results. The conclusion and some suggestions for future research are presented in Section 5. Procedure for Paper Submission.

## II. PROBLEM STATEMENT

There are  $N$  products with a particular brand, all supposed to be seasonal products with a limited life time. As mentioned before, fashion branded clothing products can be an example. It is assumed that all these products have the same life time, called *time horizon*. The time horizon is divided into *sale periods*. Advertisements are divided into two categories, *brand* and *product advertisements*. It is assumed the brand advertisement affects the demand of all products. However, the advertisement for each product can only affect the demand of that product as well as the price that customers are willing to pay for it. In each sale period, the demand for each product follows a nonhomogeneous Poisson process, which depends on the price and advertisement.

The objective is to maximize the total revenue of the company. To reach this goal, at the beginning of each sale period the decision maker has to determine:

- the optimal price of each product,
- the appropriate brand advertising,
- the appropriate advertising package for each product.

### A. Reservation Price

The maximum amount that a customer is willing to pay for purchasing one unit of a product is called "*reservation price*" of that customer. Obviously, a customer purchases a product only if his/her reservation price is higher than the price set for that product. On the other hand, since different customers have different reservation prices, the reservation price of a customer, who is chosen randomly, is a random variable. Therefore, let define,

$F_{t,i}(\cdot)$ : the cumulative distribution function of the reservation price of customers who arrive in period  $t$  to purchase one unit of product  $i$ .

### B. Notations

$N$	The number of products with the same brand
$C_i$	The inventory level of product $i$ at the beginning of time horizon
$C$	The set of inventory level of all the products, $C=\{C_1, C_2, \dots, C_N\}$
$\lambda_{t,i}$	The demand rate for product $i$ in sale period $t$
$\Lambda_t$	The set of demand rates in sale period $t$ , $\Lambda_t = \{\lambda_{t,1}, \lambda_{t,2}, \dots, \lambda_{t,N}\}$
$K_i$	The salvage value of product $i$ at the end of time horizon.
$K$	The set of salvage value of all the product, $K=\{K_1, K_2, \dots, K_N\}$
$SP_i$	The set of possible prices for product $i$ .
$RA_i^k$	Package $k$ to advertise product $i$
$r_i^s$	Cost of advertising package $RA_i^k$
$RA_i$	The set of different advertising packages for product $i$ , $RA_i = \{RA_i^1, RA_i^2, \dots, RA_i^S\}$
$F_{t,i}^s(\cdot)$	The cumulative distribution function of reservation price of customers of product $i$ in period $t$ , if advertising package $RA_i^k$ is selected to promote this product type in that period.
$BA$	The set of packages to promote brand, $BA=\{B_1, B_2, \dots, B_V\}$
$b_l$	Cost of advertising package $B_l \in BA$
$\rho_l$	The multiplication factor of demand rate for all products, if advertising package $B_l$ is selected to promote the brand.

### C. The Assumptions

- The price of product  $i$  in each sale period should be selected from  $SP_i$ . This set is defined by the experts in advance.
- The advertising options are considered as different packages. Each package may include one or several methods of advertising with different costs. Each package has a known cost and predictable effect on the reservation price (and consequently on the rate of demand). These alternative packages should be prepared based on the experience of salesmen and other logical considerations.
- The appropriate advertising package for each product and for the brand should be selected from predefined sets  $RA_i$  and  $BA$ , respectively. These sets are also predefined by experts, the same as  $SP_i$ .
- The selected package to advertise product  $i$  affects its reservation price.
- The selected package to advertise the brand affects the demand rate for that brand.

### III. MODELING THE PROBLEM

In this section, first we develop a model within the framework of stochastic dynamic programming and the concept of dynamic pricing, called “integrated dynamic pricing method”. However, this approach cannot be practical for large scale problems due to the computational complexity. Therefore, we also develop another method for larger problems which results in near optimal solution, called “hierarchical method”.

#### A. Integrated Dynamic Pricing Method

We develop an integrated dynamic pricing model which represents all products and the brand altogether. The elements of the dynamic programming model are as follows.

- Stage: sale periods, indexed by  $t$  and run backward. Thus,  $t=0$  represents the end of time horizon.
- State: The set of inventory levels,  $C$ .
- Decision Variables: Suitable price for each product, appropriate advertising package for each product as well as for the brand.

Let define  $V_t(C)$  as the maximal expected revenue generated from period  $t$  till the end of time horizon, provided the inventory is  $C$ .

Without loss of generality and for the sake of simplicity, the dynamic pricing model is presented for two products, as follows.

$$\begin{aligned}
 V_t(C_1, C_2) = & \max_{SP, RA, BA} \left\{ \sum_{j=0}^{C_1} [\Pr\{x_1(P_Z) = j\}] * \right. \\
 & \left( \sum_{i=0}^{C_2} [\Pr\{x_2(P_Z) = i\}] * (j * P_{Z_1} + i * P_{Z_2} + V_{t-1}(C_1 - j, C_2 - i)) \right) \\
 & + \sum_{i=C_2+1}^{\infty} [\Pr\{x_2(P_Z) = i\}] * \{j * P_{Z_1} + C_2 * P_{Z_2} + V_{t-1}(C_1 - j, 0)\} \\
 & + \sum_{j=C_1+1}^{\infty} [\Pr\{x_1(P_Z) = j\}] * \left( \sum_{i=0}^{C_2} [\Pr\{x_2(P_Z) = i\}] * \right. \\
 & \left. (C_1 * P_{Z_1} + i * P_{Z_2} + V_{t-1}(0, C_2 - i)) \right) + \sum_{i=C_2+1}^{\infty} [\Pr\{x_2(P_Z) = i\}] * \\
 & \left. [C_1 * P_{Z_1} + C_2 * P_{Z_2} + V_{t-1}(0, 0)] - r_1^S - r_2^S - b_v \right\}
 \end{aligned} \tag{1}$$

Boundary Equations:

$$V_0(C_1, C_2) = K_1 * C_1 + K_2 * C_2 \tag{2}$$

$$q_{t,i}(P_z) = \int_{t_{i-1}}^{t_i} \rho_v \lambda_{t,i} (1 - F_{t,i}^s(P_z)) dt \tag{3}$$

$$\Pr_t \{x_i(P_z) = j\} = \exp\{-q_{t,i}(P_z)\} * \frac{q_{t,i}(P_z)^j}{j!} \tag{4}$$

where,  $p_{z_i}$  is the selected price for product  $i$ .

This model can be solved by backward dynamic programming approach. For problems with modest size this approach works efficiently. However, as mentioned before, it is not efficient for problems with many products and sale periods. Therefore, we develop another approach.

#### B. Hierarchical Method

In this approach, in each iteration two different models are solved sequentially. We show that the total revenue will be increased after each iteration. The first model is solved  $N$  times, each time to identify the appropriate price and advertisement package for each product. In the second model, it is assumed that all products are the same. By solving this model, the suitable package for advertising the brand of the company is determined. Now, we introduce the models which should be solved sequentially.

##### 1. First Model

This is a dynamic programming model which is applied for product  $i$ .

Let  $V_{t,i}(C_i)$  represent the maximal expected revenue of products  $i$  generated from period  $t$  till the end of sale horizon, provided the state of system is  $C_i$ , without considering the brand advertisement.

$$\begin{aligned}
 V_{t,i}(C_i) = & \max_{SP_i, RA_i} \left\{ \sum_{j=0}^{C_i} \{\Pr\{x_i(P_Z) = j\} * \right. \\
 & (jP_Z + V_{t-1,i}(C_i - j))\} + \sum_{j=C_i+1}^{\infty} \Pr\{x_i(P_Z) = j\} * \\
 & \left. (C_i P_Z + V_{t-1,i}(0)) - r_i^S \right\}
 \end{aligned} \tag{5}$$

Boundary Equations:

$$V_{0,i}(C_i) = K_i C_i \tag{6}$$

$$\Pr_t \{x_i(P_z) = j\} = \exp\{-q_{t,i}(P_z)\} * \frac{q_{t,i}(P_z)^j}{j!} \tag{7}$$

$$q_{t,i}(P_z) = \int_{t_{i-1}}^{t_i} \lambda_{t,i} (1 - F_{t,i}^s(P_z)) dt \tag{8}$$

The parameter and variables are the same as previous ones. As mentioned before, the demand is supposed to follow a Poisson process. This model is solved for each product separately, which results in determining the appropriate price and advertisement package for products.

##### 2. Second Model

The goal of this model is to choose the suitable advertising package for the brand. To simplify the model, we suppose that all products are the same. This model is simpler than the first

one by assuming that the prices are known and we are just looking for a suitable advertising package. Adopting this assumption requires to set a single price, demand rate and cumulative distribution function of reservation price for all products. Let this unique price, arrival rate and cumulative probability distribution of reservation price at the beginning of sale period t be  $\tilde{P}_t$ ,  $\tilde{\lambda}_t$  and  $\tilde{F}_t$ , respectively.  $\tilde{C}$  is also the inventory level of the assumed product.  $\tilde{P}_t$ ,  $\tilde{\lambda}_t$  and  $\tilde{C}$  are figured out as follows.

$$\tilde{\lambda}_t = \sum_{i=1}^N \lambda_{t,i}^* \quad (9)$$

$$\tilde{P}_t = \frac{\sum_{i=1}^N (P_{t,i}^* * C_i)}{\sum_{i=1}^N C_i} \quad (10)$$

$$\tilde{C} = \sum_{i=1}^N C_i \quad (11)$$

As mentioned before, cost of  $b_1$  occurs if the advertising package  $B_1$  is selected. Furthermore, selecting this package causes the demand rate of all products be multiplied by  $\rho_1$ . Therefore, we have:

$$\begin{aligned} \Pr_{t,i}\{x(\tilde{P}_t) = j\} &= \exp\{-q_t(\tilde{P}_t)\} * \frac{q_t(\tilde{P}_t)^j}{j!} * [\exp(\rho_1^i) * \rho_1^j] \\ &= \Pr_{t,i}\{x(\tilde{P}_t) = j\} * [\exp(\rho_1^i) * \rho_1^j] \end{aligned} \quad (12)$$

$$\Pr_{t,i}\{x(\tilde{P}_t) = j\} = \exp\{-q_t(\tilde{P}_t)\} * \frac{q_t(\tilde{P}_t)^j}{j!} \quad (13)$$

$$q_t(\tilde{P}_t) = \int_{t_{i-1}}^{t_i} \tilde{\lambda}_t (1 - \tilde{F}_t(\tilde{P}_t)) dt \quad (14)$$

As a result,

$$\begin{aligned} W_t(\tilde{C}) &= \max_{BA} \left\{ \sum_{j=0}^{\tilde{C}} \{ \Pr_{t,i}\{x(\tilde{P}_t) = j\} * \right. \\ &\quad \left. (j\tilde{P}_t + W_{t-1}(\tilde{C} - j)) \} + \sum_{j=\tilde{C}}^{\infty} \{ \Pr_{t,i}\{x(\tilde{P}_t) = j\} * \right. \\ &\quad \left. (\tilde{C}\tilde{P}_t + W_{t-1}(0)) - b_1 \} \right\} \end{aligned} \quad (15)$$

Considering that the price in this model is pre-specified, selecting a package only affects  $\rho_1$ . We can simplify the model as follows.

$$\begin{aligned} W_t(\tilde{C}) &= \max_{BA} \left\{ \sum_{j=0}^{\tilde{C}} \{ [\exp(\rho_1) * \rho_1^j] * \Pr_{t,i}\{x(\tilde{P}_t) = j\} \right. \\ &\quad \left. * (j\tilde{P}_t + W_{t-1}(\tilde{C} - j)) \} + \sum_{j=\tilde{C}}^{\infty} \{ [\exp(\rho_1) * \rho_1^j] * \right. \\ &\quad \left. \Pr_{t,i}\{x(\tilde{P}_t) = j\} * (\tilde{C}\tilde{P}_t + W_{t-1}(0)) - b_1 \} \right\} \end{aligned} \quad (16)$$

$$W_0(\tilde{C}) = \frac{\sum_{i=1}^N (K_i * C_i)}{\sum_{i=1}^N C_i} * \tilde{C} \quad (17)$$

### 3. Revenue Function

To verify the improvement of the answer in this method, we need to define a function to figure out the total revenue of the company and show that this function will increase in each iteration. Due to the dynamic aspect of the problem, it is not efficient to determine the exact amount of total revenue from time t to the end of the time horizon. In this section, we define a function, say  $Z_t$ , to estimate the total revenue from time t to the end of time horizon. we defined  $Z_t$  as follow:

$$\begin{aligned} Z_t(C, \Lambda, B_v^*, RA^*, P_{t,1}, \dots, P_{t,N}, F_1, \dots, F_N) &= \\ &= \sum_{i=1}^N X_{t,i}(C_i, \Lambda_i, RA^*, P_{t,i}, F_i) - b_v \end{aligned} \quad (18)$$

$$\begin{aligned} X_{t,i}(C_i, \Lambda_i, RA^*, P_{t,i}, F_i) &= \\ &= \sum_{j=0}^{C_i} \{ \Pr\{X_i(P_i) = j\} (jP_i + V_{t-1,i}(C_i - j)) \} + \\ &= \sum_{j=C_i+1}^{\infty} \Pr\{X_i(P_i) = j\} (C_i P_z + V_{t-1,i}(0)) - r_i^s \end{aligned} \quad (19)$$

where,

$\Lambda_i = \{\Lambda_{1,i}, \dots, \Lambda_{T,i}\}$  is the set of demand rate of product i

$\Lambda = \{\Lambda_1, \dots, \Lambda_N\}$  shows the arrival rate of all the products

$B_v^*$  is the selected advertising package for the brand

$RA^*$  shows the set of selected advertising packages for each product

$P_{t,i}$  is the price of product type i in time interval t

$F_i = \{F_{T,1}, \dots, F_{T,i}\}$  includes the cumulative distribution of reservation price of product type  $i$  in each time interval

$$Z_t(C, \Lambda'', B_v^*, RA^*, P_{1,t}', \dots, P_{N,t}', F_1', \dots, F_N') = \sum_{i=1}^N V_{t,i}(C_i; \Lambda_i'', F_i') - b_v'' \quad (23)$$

#### 4. The Algorithm

Step 1- Solve the first model for each product.

Step 2- Set  $O$  equal to  $Z_t$  (Equation 18).

Step 3- Solve the second model.

Step 4- Set  $Q$  equal to  $Z_t$  (Equation 18).

Step 5- If  $Q > O$  accept the result of solving second model and update the parameters considering the results of step 3 and go back to step 1, otherwise go to step 6.

Step 6- Remove the selected advertising package in step 3 from BA. IF BA is null, stop. Otherwise, go back to step 3.

$$V_{t,i}(C_i; \Lambda_i'', F_i') = \max_{P_{i,t}, RA_i} \{X_{t,i}(C_i, \Lambda_i'', RA_i^*, P_{i,t}'', F_i')\} \quad (24)$$

$$Z_t(M_{2,n}; C) = Z_t(\Lambda'', B_v'', RA^'', P_{1,t}'', \dots, P_{N,t}'', F_1'', \dots, F_N'') = \sum_{i=1}^N X_{t,i}(C_i, \Lambda_i'', RA_i^'', P_{i,t}'', F_i'') - b_v'' \quad (25)$$

As mentioned in the beginning of this section, we should make sure that the revenue function is improved at the end of each iteration. Suppose  $Z_t(M_{1,n}; C)$  is equal to  $Z_t$  calculated with the output parameters and variables of solving the first model, with inventory of  $C$  and in sale period  $t$ , in  $n$ th loop of algorithm. ( $n$  is the current loop number.) This function figures out at the end of step 2 in each iteration of the algorithm. We can prove that:

$$Z_t(M_{1,n+1}; C) > Z_t(M_{1,n}; C) \quad (20)$$

Proof:

To prove (20), it's required to show that:

$$Z_t(M_{2,n}; C) > Z_t(M_{1,n}; C) \quad (21)$$

$$Z_t(M_{1,n+1}; C) \geq Z_t(M_{2,n}; C) \quad (22)$$

Where  $Z_t(M_{2,n}; C)$  stands for the revenue function calculated with the output parameters figured out at the end of step 5 in the  $n$ th loop. (When the condition of fifth step is satisfied)

Due to the fifth step of the algorithm, (21) is obvious. By proving (22), we can show that the revenue will improve each time we implement the loop.

Suppose that the following parameters obtained from solving the second model (satisfying the condition stated in the fifth step) in loop no.  $n$ , and the first model in loop no.  $n+1$  respectively:

$$(\Lambda'', B_v'', RA^'', P_{1,t}'', \dots, P_{N,t}'', F_1'', \dots, F_N'')$$

$$(\Lambda', B_v', RA^*, P_{1,t}', \dots, P_{N,t}', F_1', \dots, F_N')$$

So the revenue functions are as follows

$$Z_t(M_{1,n+1}; C) =$$

In each loop we have:

$$V_{t,i}(C_i; \Lambda_i'', F_i') \geq X_{t,i}(C_i, \Lambda_i'', RA_i^*, P_{i,t}'', F_i') \quad (26)$$

Due to the algorithm,  $Z_t(M_{1,n+1}; C)$  is calculated based on the output parameters of  $V_{t,i}(C_i; \Lambda_i'', F_i')$ . Thus we can simply conclude that:

$$Z_t(M_{1,n+1}; C) = \sum_{i=1}^N V_{t,i}(C_i; \Lambda_i'', F_i') - b_v'' \geq \sum_{i=1}^N X_{t,i}(C_i, \Lambda_i'', RA_i^*, P_{i,t}'', F_i'') - b_v'' = Z_t(M_{2,n}; C) \quad (27)$$

$$Z_t(M_{1,n+1}; C) \geq Z_t(M_{2,n}; C) \quad (28)$$

Now we have proved (21) and (22), hence we can easily conclude (20).

#### IV. NUMERICAL ANALYSIS

In this section, we want to find out the accuracy of our proposed "hierarchical method" and compare it with the first method. To obtain this goal, 70 random problems with modest size are generated randomly and solved by both methods. Then, the total revenue of the company by each method is compared.

The revenue function explained in the hierarchical method is defined to be able to give us an estimation of total revenue for complicated and large problems. Considering that we are working with small problems in this part, we can develop a new revenue function which can provide us with the accurate revenue amount. The parameters of this function are as follow:

$P_{t,i}^*$  The selected price for product  $i$  in sale period  $t$

$RA_{t,i}^*$  The selected advertising package for product  $i$

in sale period t

$r_{t,i}^*$  Cost of advertising package  $RA_{i,t}^*$  for product i in sale period t

$BA_t^*$  The selected advertising package for the brand of company in sale period t

$b_t^*$  Cost of the selected advertising package for the brand of company in sale period t

$C = (C_1, C_2)$  Inventory of products in sale period t

$\lambda_{t,i}^*$  The arrival rate for products type i considering the effect of selected advertising package  $BA_t^*$

$F_{t,i}^*$  Cumulative probability distribution of the reservation price of customers who arrived to purchase one unit of type i product in period t considering the selected advertising package  $RA_{i,t}^*$ .

Considering the defined parameters, the total revenue function is as follow:

$$R_t(C, P_{t,1}^*, P_{t,2}^*, RA_{t,1}^*, RA_{t,2}^*, BA_t^*, \lambda_{t,1}^*, \lambda_{t,2}^*, F_{t,1}^*, F_{t,2}^*) = \sum_{j=0}^{C_1} [\Pr\{x_1(P_{t,1}^*) = j\} * (\sum_{i=0}^{C_2} [\Pr\{x_2(P_{t,2}^*) = i\} * (j * P_{t,1}^* + i * P_{t,2}^* + V_{t-1}(C_1 - j, C_2 - i))] + \sum_{i=C_2+1}^{\infty} [\Pr\{x_2(P_{t,2}^*) = i\} * \{j * P_{t,1}^* + C_2 * P_{t,2}^* + V_{t-1}(C_1 - j, 0)\}])] + \sum_{j=C_{1+1}}^{\infty} [\Pr\{x_1(P_{t,1}^*) = j\}] * (\sum_{i=0}^{C_2} [\Pr\{x_2(P_{t,2}^*) = i\} * (C_1 * P_{t,1}^* + i * P_{t,2}^* + V_{t-1}(0, C_2 - i))] + \sum_{i=C_2+1}^{\infty} [\Pr\{x_2(P_{t,2}^*) = i\}] * \{C_1 * P_{t,1}^* + C_2 * P_{t,2}^* + V_{t-1}(0, 0)\})] - r_{t,1}^* - r_{t,2}^* - b_t^* \quad (29)$$

Boundary Equations:

$$V_0(C_1, C_2) = K_1 * C_1 + K_2 * C_2 \quad (30)$$

$$\Pr\{x_i(P_{t,i}^*) = j\} = \exp\{-q_{t,i}(P_{t,i}^*)\} * \frac{q_{t,i}(P_{t,i}^*)^j}{j!} \quad (31)$$

$$q_{t,i}(P_{t,i}^*) = \int_{t_{i-1}}^{t_i} \lambda_{t,i}^* (1 - F_{t,i}^*(P_{t,i}^*)) dt \quad (32)$$

The revenue function is calculated for 70 different random problems. Then, the following function is figured out for each problem:

Ratio Function = (Total revenue of the outputs of Hierarchical Method)/(Total revenue of the outputs of integrated Method)

The average and variance of the defined ratio function calculated for the 70 problems are 0.8 and 0.06 respectively. Fig. 1 presents the amount of Ratio Function for all the defined problems.

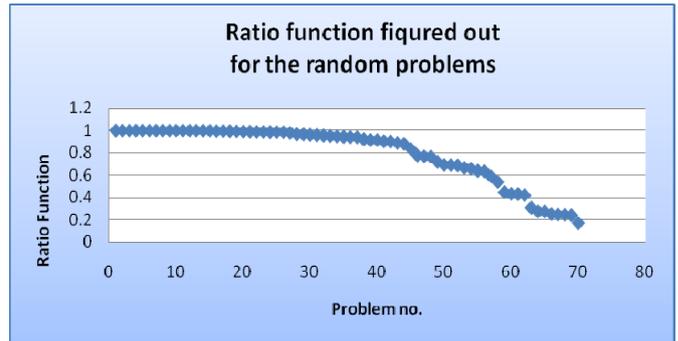


Fig. 1 Ratio Function

#### V. CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

In this paper two different methods are presented to find out simultaneously the suitable prices and advertising methods of seasonal products. The advertisements are classified into two different groups. to advertise the products or to advertise the brand of the company. It is obvious the advertising methods selected for different products a not be necessarily the same.

The first method presented is an integrated dynamic pricing model which can find the optimal price of each product as well the suitable packages for products and the brand simultaneously. Since applying dynamic programming for large sized problems is time consuming and mostly inefficient, the integrated model is recommended for the problems with modest size.

The second method presented in this paper is capable of finding the solution with good accuracy in a timely manner for real world problems, although the solution is not necessarily optimal. However, the numerical examples showed the solutions are mostly close to optimal.

In the second method suggested in this paper, there is a probability that we stuck in the local optimum point. This probability can be decreased with running the suggested algorithm for different times, each time with a different starting amount.

One direction for future researches is to extend the second method in this way or with other methods to decrease the probability and increase the accuracy of the result. The other suggestion is to extend the problem considering the suitable amount of productions as decision variables or considering e-learning in the models. As such, since this is the first time in the literature of Revenue management and pricing that this problem (considering the effect of pricing and advertising on

each other) is discussed, the suggested methods can be an encouraging first step toward the future achievements and researches.

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