World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:3, No:8, 2009

On the Numbers of Various Young Tableaux

Hsuan-Chu Li

***Abstract*—**We demonstrate a way to count the number of Young tableau of shape $\lambda=(k,k,\cdots,k)$ with $|\lambda|=lk$ by expanding Schur function. This result gives an answer to the question that was put out by Jenny Buontempo and Brian Hopkins.

***Keywords*—**Young tableau, Schur function.

I. INTRODUCTION

Given a partition $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_n)$ of p, a semi-standard Young tableau of shape λ $SSYT^{\lambda}$ is an arrangement of p boxes, λ_1 in the first row, λ_2 in the second row sharing left border with the first row, etc., with each boxes having a label from $\{1,2,\cdots,p\}$ such that labels weakly increasing across rows and strictly increasing down columns. If the labels increase across rows and down columns, then it becomes a Young tableau of shape λ. The following two examples show a $SSYT^{5221}$ and a Young tableau of shape $(5,2,2,1)$ respectively.

1	2	2	2	5
2	4			
3	6			
5				

1	3	5	8	10
2	4			
6	9			
7				

Let

$$G_{\{n;a_1,a_2,\cdots,a_n\}}=\begin{bmatrix} x_1^{a_1} & x_1^{a_2} & \cdots & x_1^{a_n} \\ x_2^{a_1} & x_2^{a_2} & \cdots & x_2^{a_n} \\ \vdots & \vdots & \ddots & \vdots \\ x_n^{a_1} & x_n^{a_2} & \cdots & x_n^{a_n} \end{bmatrix}$$

are integers and $0<x_1<x_2<\cdots x_n$ [1, p.142]. In connection with Schur functions we define the partition λ associated with G as the nonincreasing sequence of nonnegative integers

$$\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_n)=(a_n-(n-1),a_{n-1}-(n-2),\cdots a_1),$$

Hsuan-Chu Li is with the Center for General Education, Jen-Teh Juinor College of Medicine, Nursing and Management,Miaoli, Taiwan, as Assistant Professor. e-mail: k0401001@ms4.kntech.com.tw.

and get

$$G_{\{n;a_1,a_2,\cdots a_n\}}=[x_i^{j-1+\lambda_{n-j+1}}]_{1\le i,j\le n},$$

and Schur function associated to λ is defined as

$$s_{\lambda}(x_1,x_2,\cdots x_n)=\frac{\det G_{\{n;a_1,a_2,\cdots a_n\}}}{\det G_{\{n;0,1,\cdots n-1\}}}.$$

In [2], Jenny Buontempo and Brian Hopkins considered Young tableaux consisting of two rows of equal length counted by the Catalan numbers and gave two combinatorial proofs.

In a recent paper [3], we succeeded to provide the unique LU factorization of $G_{\{n;a,a_2,\cdots a_n\}}$ avoiding Schur function and expressed any Schur function in an explicit form. Now we state the results as follows:

1. G_n can be factorized as $G_n=L_nU_n$, where $L_n=[L_n(i,j)]$ is a lower triangular matrix with unit main diagonal and $U_n=[U_n(i,j)]$ is an upper triangular matrix, whose entries are defined as follows:

$$L_n(i,j)=\begin{cases} 1, i=j; \\ 0, i<j; \\ (\frac{x_i}{x_1})^{a_1}, j=1, i\ge 2; \\ (\frac{x_i}{x_j})^{a_1}\frac{S_{\{x_j\to x_i\}}A_j}{A_j}, i\ge j+1, j\ge 2. \end{cases}$$

and

$$U_n(i,j)=\begin{cases} x_1^{a_j}, i=1; \\ 0, i>j; \\ x_i^{a_1}B_i, i=j\ge 2; \\ x_i^{a_1}S^{\{a_i\to a_j\}}(B_i), j\ge i+1, i\ge 2. \end{cases}$$

where

$$A_2=B_2=x_2^{a_2-a_1}-x_1^{a_2-a_1};$$

$$A_k=\{S^{\{a_{k-1}\to a_k\}}_{\{x_{k-1}\to x_k\}}(A_{k-1})\}A_{k-1}-S_{\{x_{k-1}\to x_k\}}(A_{k-1})S^{\{a_{k-1}\to a_k\}}(A_{k-1}), k\ge 3; \quad (1.1)$$

$$A_k=S^{\{a_{k-1}\to a_k\}}_{\{x_{k-1}\to x_k\}}(B_{k-1})-\{S^{\{a_{k-1}\to a_k\}}(B_{k-1})\}(\frac{S_{\{x_{k-1}\to x_k\}}(A_{k-1})}{A_{k-1}}), k\ge 3; \quad (1.2)$$

$$S_{\{x_{k-1}\to x_k\}}(A_{k-1}):=x_k \textit{ substitutes for } x_{k-1} \textit{ in } A_{k-1};$$

$$S^{\{a_{k-1}\to a_k\}}(A_{k-1}):=a_k \textit{ substitutes for } a_{k-1} \textit{ in } A_{k-1}.$$

2. The Schur function $s_\lambda(x_1, x_2, \cdots x_n)$ can be expressed as

$$s_\lambda(x_1, x_2, \cdots x_n) = \frac{x_1^{a_1} \times \prod_{2\le i\le n} x_i^{a_1} B_i}{\prod_{1\le i<j\le n}(x_j - x_i)}$$

where B_i are defined as above.

Basing on the above facts, we establish a method to count the number of various Young tableaux, including those with more than 2 rows, while remaining within a tableaux context.

II. MAIN RESULT AND AN EXAMPLE

Now we are in a position to state the main theorem.

Theorem II.1 The number of Young tableau of shape $\lambda = (k, k, \cdots, k)$ with $|\lambda| = lk$ is the coefficient of the term $x_1 x_2 \cdots x_{lk}$ in the expansion of $s_{(k,k,\cdots,k)}(x_1, x_2, \cdots, x_{lk})$.

Proof. By definition, the Schur function s_λ is the symmetric function defined as:

$$s_\lambda(x_1, x_2, \cdots x_{lk}) = \sum_{T\in SSYT^\lambda} X^T = \sum_{T\in SSYT^\lambda} x_1^{m_1} x_2^{m_2} \cdots x_{lk}^{m_k}$$

where $|\lambda| = lk$, m_i is the number of entries i in $SSYT^\lambda$ T for $i = 1, 2, \cdots, lk$. In particular, when $(m_1, m_2, \cdots, m_{lk}) = (1, 1, \cdots, 1)$, then the coefficient of the term $x_1 x_2 \cdots x_{lk}$ is exactly the number of Young tableau of shape $\lambda = (k, k, \cdots, k)$ with $|\lambda| = lk$.

Now let us take a look at the following example to see how to count the number of Young tableau of shape λ.

Example II.2 By using the recurrence formula (1.1) and (1.2), we can get

$$A_3 = (x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1});$$

$$A_4 = [(x_4^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_3-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]];$$

$$A_5 = \{[(x_5^{a_5-a_1} - x_1^{a_5-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_5-a_1} - x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1} - x_1^{a_5-a_1})(x_2^{a_3-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_5-a_1} - x_1^{a_5-a_1})]]\}$$
$$\times\{[(x_4^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_3-a_1} - x_1^{a_2-a_1})$$
$$-[(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]]\}$$
$$-\{[(x_5^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1} - x_1^{a_4-a_1})(x_2^{a_3-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_4-a_1} - x_1^{a_4-a_1})]]\}$$
$$\times\{[(x_4^{a_5-a_1} - x_1^{a_5-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_5-a_1} - x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1} - x_1^{a_5-a_1})(x_2^{a_3-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_5-a_1} - x_1^{a_5-a_1})]]\};$$

$$A_6 = \{\{[(x_6^{a_6-a_1} - x_1^{a_6-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_6-a_1} - x_1^{a_6-a_1})]$$
$$\times[(x_3^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1} - x_1^{a_3-a_1})(x_2^{a_2-a_1} - x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1} - x_1^{a_2-a_1})(x_2^{a_3-a_1} - x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$-\{[(x_6^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$\times\{[(x_4^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]\}\}$$
$$\times\{\{[(x_5^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$-\{[(x_5^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$\times\{[(x_4^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]\}\}$$
$$-\{\{[(x_6^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$-\{[(x_6^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$\times\{[(x_4^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]\}\}$$
$$\times\{\{[(x_5^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$
$$-\{[(x_5^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_{54}-a_1}-x_1^{a_4-a_1})]\}$$
$$\times\{[(x_4^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_6-a_1}-x_1^{a_6-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_6-a_1}-x_1^{a_6-a_1})]\}\};$$

$$B_3=(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}];$$

$$B_4=\{(x_4^{a_4-a_1}-x_1^{a_4-a_1})-(x_2^{a_4-a_1}-x_1^{a_4-a_1})\times[\frac{(x_4^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$
$$-\{(x_3^{a_4-a_1}-x_1^{a_4-a_1})-(x_2^{a_4-a_1}-x_1^{a_4-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$
$$\times\{\frac{(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$
$$\frac{-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\};$$

$$B_5=\{(x_5^{a_5-a_1}-x_1^{a_5-a_1})-(x_2^{a_5-a_1}-x_1^{a_5-a_1})\times[\frac{(x_5^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$
$$-\{(x_3^{a_5-a_1}-x_1^{a_5-a_1})-(x_2^{a_5-a_1}-x_1^{a_5-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$
$$\times\{\frac{(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$
$$\frac{-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$
$$-\{\{(x_4^{a_5-a_1}-x_1^{a_5-a_1})-(x_2^{a_5-a_1}-x_1^{a_5-a_1})\times[\frac{(x_4^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$-\{(x_3^{a_5-a_1}-x_1^{a_5-a_1})-(x_2^{a_5-a_1}-x_1^{a_5-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$\times\{\frac{(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$

$$\frac{-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$

$$\times\{\{[(x_5^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$

$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_3-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$

$$\div\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$

$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}\};$$

$$B_6=\{\{\{(x_6^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_6^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$-\{(x_3^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$\times\{\frac{(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$

$$\frac{-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$

$$-\{\{(x_4^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_4^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$-\{(x_3^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$\times\{\frac{(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$

$$\frac{-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$

$$\times\{\{[(x_6^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$

$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}$$

$$\div\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$

$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$

$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]\}\}\}$$

$$-\{\{\{(x_5^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_5^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$-\{(x_3^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$\times\{\frac{(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$

$$\frac{-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$

$$-\{\{(x_4^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_4^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$-\{(x_3^{a_6-a_1}-x_1^{a_6-a_1})-(x_2^{a_6-a_1}-x_1^{a_6-a_1})\times[\frac{(x_3^{a_2-a_1}-x_1^{a_2-a_1})}{(x_2^{a_2-a_1}-x_1^{a_2-a_1})}]\}$$

$$\times\{\frac{(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}{(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})}$$

$$\frac{-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}{-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})}\}\}$$

$$\times\{\{[(x_5^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$

$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}$$
$$\div\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}\}\}$$
$$\times\{\{\{[(x_6^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}$$
$$-\{[(x_6^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_6^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_6^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}$$
$$\times\{[(x_4^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]]\}\}$$
$$\div\{\{[(x_5^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]]\}$$
$$\times\{[(x_4^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}$$
$$-\{[(x_5^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_5^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_5^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_4-a_1}-x_1^{a_4-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_4-a_1}-x_1^{a_4-a_1})]]\}$$
$$\times\{[(x_4^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]$$
$$\times[(x_3^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$

$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$-[(x_4^{a_3-a_1}-x_1^{a_3-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_4^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_3-a_1}-x_1^{a_3-a_1})]$$
$$\times[(x_3^{a_5-a_1}-x_1^{a_5-a_1})(x_2^{a_2-a_1}-x_1^{a_2-a_1})$$
$$-(x_3^{a_2-a_1}-x_1^{a_2-a_1})(x_2^{a_5-a_1}-x_1^{a_5-a_1})]]\}\}.$$

Now considering $\lambda=(2,2,2)=(2,2,2,0,0,0)$, then $(a_1,a_2,a_3,a_4,a_5,a_6)=(0,1,2,5,6,7)$ and

$$s_{(2,2,2)}(x_1,x_2,x_3,x_4,x_5,x_6)$$
$$=\frac{\det G_{\{6;0,1,2,5,6,7\}}}{\prod\limits_{1\le i,j\le 6}(x_j-x_i)}=\frac{\prod\limits_{2\le i\le 6}B_i}{\prod\limits_{1\le i,j\le 6}(x_j-x_i)}$$
$$=x_1^2x_2^2x_3^2+x_1^2x_2^2x_3x_4+x_1^2x_2x_3^2x_4+x_1x_2^2x_3^2x_4$$
$$+x_1^2x_2^2x_4^2+x_1^2x_2x_3x_4^2+x_1x_2^2x_3x_4^2+x_1^2x_3^2x_4^2$$
$$+x_1^2x_2x_3^2x_4^2+xx_2^2x_3^2x_4^2+x_1^2x_2^2x_3x_5+x_1^2x_2x_3^2x_5$$
$$+x_1x_2^2x_3^2x_5+x_1^2x_2^2x_4x_5+2x_1^2x_2x_3x_4x_5+2x_1x_2^2x_3x_4x_5$$
$$+x_1^2x_3^2x_4x_5+2x_1x_2x_3^2x_4x_5+x_2^2x_3^2x_4x_5+x_1^2x_2x_4^2x_5$$
$$+x_1x_2^2x_4^2x_5+x_1^2x_3x_4^2x_5+2x_1x_2x_3x_4^2x_5+x_2^2x_3x_4^2x_5$$
$$+x_1x_3^2x_4^2x_5+x_2x_3^2x_4^2x_5+x_1^2x_2^2x_5^2+x_1^2x_2x_3x_5^2$$
$$+x_1x_2^2x_3x_5^2+x_1^2x_3^2x_5^2+x_1x_2x_3^2x_5^2+x_2^2x_3^2x_5^2$$
$$+x_1^2x_2x_4x_5^2+x_1x_2^2x_4x_5^2+x_1^2x_3x_4x_5^2+2x_1x_2x_3x_4x_5^2$$
$$+x_2^2x_3x_4x_5^2+x_1x_3^2x_4x_5^2+x_2x_3^2x_4x_5^2+x_1^2x_4^2x_5^2$$
$$+x_1x_2x_4^2x_5^2+x_2^2x_4^2x_5^2+x_1x_3x_4^2x_5^2+x_2x_3x_4^2x_5^2$$
$$+x_3^2x_4^2x_5^2+x_1^2x_2^2x_3x_6+x_1^2x_2x_3^2x_6+x_1x_2^2x_3^2x_6$$
$$+x_1^2x_2^2x_4x_6+2x_1^2x_2x_3x_4x_6+2x_1x_2^2x_3x_4x_6+x_1^2x_3^2x_4x_6$$
$$+2x_1x_2x_3^2x_4x_6+x_2^2x_3^2x_4x_6+x_1^2x_2x_4^2x_6+x_1x_2^2x_4^2x_6$$
$$+x_1^2x_3x_4^2x_6+2x_1x_2x_3x_4^2x_6+x_2^2x_3x_4^2x_6+x_1x_3^2x_4^2x_6$$
$$+x_2x_3^2x_4^2x_6+x_1^2x_2^2x_5x_6+2x_1^2x_2x_3x_5x_6+2x_1x_2^2x_3x_5x_6$$
$$+x_1^2x_3^2x_5x_6+2x_1x_2xx_3^2x_5x_6+x_2^2x_3^2x_5x_6+2x_1^2x_2x_4x_5x_6$$
$$+2x_1x_2^2x_4x_5x_6+2x_1^2x_3x_4x_5x_6+5x_1x_2x_3x_4x_5x_6+2x_2^2x_3x_4x_5x_6$$
$$+2x_1x_3^2x_4x_5x_6+2x_2x_3^2x_4x_5x_6+x_1^2x_4^2x_5x_6+2x_1x_2x_4^2x_5x_6$$
$$+x_2^2x_4^2x_5x_6+2x_1x_3x_4^2x_5x_6+2x_2x_3x_4^2x_5x_6+x_3^2x_4^2x_5x_6$$
$$+x_1^2x_2x_5^2x_6+x_1x_2^2x_5^2x_6+x_1^2x_3x_5^2x_6+2x_1x_2x_3x_5^2x_6$$
$$+x_2^2x_3x_5^2x_6+x_1x_3^2x_5^2x_6+x_2x_3^2x_5^2x_6+x_1^2x_4x_5^2x_6$$
$$+2x_1x_2x_4x_5^2x_6+x_2^2x_4x_5^2x_6+2x_1x_3x_4x_5^2x_6+2x_2x_3x_4x_5^2x_6$$
$$+x_3^2x_4x_5^2x_6+x_1x_4^2x_5^2x_6+x_2x_4^2x_5^2x_6+x_3x_4^2x_5^2x_6$$
$$+x_1^2x_2^2x_6^2+x_1^2x_2x_3x_6^2+x_1x_2^2x_3x_6^2+x_1^2x_3^2x_6^2$$
$$+x_1x_2x_3^2x_6^2+x_2^2x_3^2x_6^2+x_1^2x_2x_4x_6^2+x_1x_2^2x_4x_6^2$$
$$+x_1^2x_3x_4x_6^2+2x_1x_2x_3x_4x_6^2+x_2^2x_3x_4x_6^2+x_1x_3^2x_4x_6^2$$
$$+x_2x_3^2x_4x_6^2+x_1^2x_4^2x_6^2+x_1x_2x_4^2x_6^2+x_2^2x_4^2x_6^2$$
$$+x_1x_3x_4^2x_6^2+x_2x_3x_4^2x_6^2+x_3^2x_4^2x_6^2+x_1^2x_2x_5x_6^2$$
$$+x_1x_2^2x_5x_6^2+x_1^2x_3x_5x_6^2+2x_1x_2x_3x_5x_6^2+x_2^2x_3x_5x_6^2$$
$$+x_1x_3^2x_5x_6^2+x_2x_3^2x_5x_6^2+x_1^2x_4x_5x_6^2+2x_1x_2x_4x_5x_6^2$$
$$+x_2^2x_4x_5x_6^2+2x_1x_3x_4x_5x_6^2+2x_2x_3x_4x_5x_6^2+x_3^2x_4x_5x_6^2$$
$$+x_1x_4^2x_5x_6^2+x_2x_4^2x_5x_6^2+x_3x_4^2x_5x_6^2+x_1^2x_5^2x_6^2$$
$$+x_1x_2x_5^2x_6^2+x_2^2x_5^2x_6^2+x_1x_3x_5^2x_6^2+x_2x_3x_5^2x_6^2$$
$$+x_3^2x_5^2x_6^2+x_1x_4x_5^2x_6^2+x_2x_4x_5^2x_6^2+x_3x_4x_5^2x_6^2$$
$$+x_4^2x_5^2x_6^2.$$

Observing that the coefficient of the term $x_1x_2x_3x_4x_5x_6$ is 5, which is exactly the number of Young tableau of shape $\lambda=(2,2,2)$.

REFERENCES

[1] J. Demmel and P. Koev, The accurate and efficient solution of a totally positive generalized Vandermonde linear system, SIAM J. Matrix Anal. Appl., 27(1)(2005), 142-152.

[2] Jenny Buontempo, Brian Hopkins, Tableau Cycling and Catalan Numbers, INTEGERS: Electronic Journal of Combinatorial Number Theory 7 (2007), #A45.

[3] Young-Ming Chen, Hsuan-Chu Li and Eng-Tjioe Tan, An Explicit Factorization of Totally Positive Generalized Vandermonde Matrices Avoiding Schur Functions, Applied Mathematics E-Notes, 8(2008), 138-147.