

# Mechanical Characteristics on Fatigue Crack Propagation in Aluminium Plate

A. Chellil, A. Nour, S. Lecheb, H. Mechakra, L. Addar, H. Kebir

**Abstract**—This paper present a mechanical characteristics on fatigue crack propagation in Aluminium Plate based on strain and stress distribution using the abaqus software. The changes in shear strain and stress distribution during the fatigue cycle with crack growth is identified. In progressive crack in the strain distribution and the stress is increase in the critical zone. Numerical Modal analysis of the model developed, prove that the Eigen frequencies of aluminium plate were decreased after cracking, and this reduce is nonlinear. These results can provide a reference for analysts and designers of aluminium alloys in aeronautical systems.

Therefore, the modal analysis is an important factor for monitoring the aeronautic structures.

**Keywords**—Aluminium alloys, plate, crack, failure.

## I. INTRODUCTION

THE preservation of structures against fatigue and fracture, is essential to characterize and control their vibrational behavior. One cause of failure is structural fatigue due to the magnitude of the vibration amplitudes in response to a deterministic or random forced excitation, adaptive materials seem an ideal solution thanks to their integrated multifunctionality. We are very interested in many industrial sectors such as aeronautics, aerospace, shipbuilding, automobile construction, appliances, civil engineering, nuclear engineering, acoustics and many other. To reduce and control the amplitude of vibration, three types of solutions exist.

In the work of Shuyuan Zhang [1], a mathematical model was developed to calculate the residual stress in a thick aluminum plate. It is based on the equilibrium stress of plasticity in two dimensions and design free size. The model is checked against the published experiments on aluminum plate.

Analytical models for thermo-structural failure during fire have been reported in the literature. Models based on buckling behavior have been proposed [2] with limited validation.

An analytical failure criterion is proposed to characterise ship plated structures manufactured with aluminium or steel materials subjected to low impact velocities [3]. The criterion considers the critical deflection, force and absorbed energy of plates laterally impacted by a hemispherical indenter, and assumes that failure occurs at the presence of necking. The

proposed expressions are compared with numerical results validated with drop weight experiments conducted on small-scaled rectangular aluminium and steel plates of the same bending stiffness.

Suzuki et al. [4] developed empirical correlations to predict failure temperatures of beams and columns, but was limited to validation with data from fire resistance tests.

Some authors have proposed complementary tests using more complex test setups in combination with inverse methods [5]-[7]. These tests can include complex loading conditions and complex geometries of the plate. The produced heterogeneity in the deformation fields can more efficiently approximate the real life situation. Consequently, the material models obtained from these tests will be more suitable for numerical simulations.

The aim of this article is to identify the strain and stress behavior of the aluminum plates subjected to damage growth and dynamic characteristic for the plate.

## II. MATHEMATICAL MODEL OF THE PLATE

### A. Kinematic Modeling (Displacement Field)

In the theory of plates, the behavior of the points of the plate is reduced to that of the medium surface and a displacement field is assumed as the variable (variable limited by development).

$$u(x, y, z) = u(x, y, 0) + z\varphi_1(x, y) + z^2\psi_1(x, y) + z^3\phi_1(x, y)$$

$$v(x, y, z) = v(x, y, 0) + z\varphi_2(x, y) + z^2\psi_2(x, y) + z^3\phi_2(x, y) \quad (1)$$

$$w(x, y, z) = w(x, y, 0) + z\varphi_3(x, y) + z^2\psi_3(x, y)$$

As is still small compared to other dimensions, it is assumed that a pattern of first degree is sufficient. The displacement field of a point M0 is denoted by a result one of the following notations:

$$u_0 = u_0(x, y, 0) = u(x, y)$$

$$v_0 = v_0(x, y, 0) = u(x, y) \quad (2)$$

$$w_0 = w_0(x, y, 0) = u(x, y)$$

### B. The Strain Field

The strain is deduced from the displacement field (2) by:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z \frac{\partial \varphi_x}{\partial x}$$

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$$\begin{aligned} \varepsilon_{yy} &= \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} + z \frac{\partial \varphi_y}{\partial y} \\ \varepsilon_{zz} &= \frac{\partial w}{\partial z} = \frac{\partial w_0}{\partial z} = 0 \\ \gamma_{xy} &= 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + z \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left( \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \right) \\ \gamma_{xz} &= 2\varepsilon_{xz} = \frac{\partial w}{\partial x} + z \frac{\partial u}{\partial z} = \frac{\partial w_0}{\partial x} + \varphi_x \\ \gamma_{yz} &= 2\varepsilon_{yz} = \frac{\partial w}{\partial y} + z \frac{\partial v}{\partial z} = \frac{\partial w_0}{\partial y} + \varphi_y \end{aligned} \quad (3)$$

This strain field is that a scheme of first degree transverse shear, it is the result of two contributions: membrane strain and bending strain and torsion.

### C. The Stress Field

The plate theory is intended to simplify the problem of three dimensional (x, y, z) in a two-dimensional problem (x, y). The reduction of the problem is obtained by integrating along the thickness.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{16} & Q_{26} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} \\ 0 & 0 & 0 & C_{45} & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{zz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (4)$$

$$\varepsilon_{zz} = \frac{1}{C_{33}} (C_{13}\varepsilon_{xx} + C_{23}\varepsilon_{yy} + C_{36}\gamma_{xy}) \quad (5)$$

In the plane of the plate, the normal and shear forces are:

$$N(x, y) = \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \sum_1^n \int_{h_{k-1}}^{h_k} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}^k dz \quad (6)$$

In shear:

$$Q(x, y) = \begin{pmatrix} Q_x \\ Q_y \end{pmatrix} = \sum_1^n \int_{h_{k-1}}^{h_k} \begin{pmatrix} \sigma_{xz} \\ \sigma_{yz} \end{pmatrix}^k dz \quad (7)$$

Similarly, we define the bending and torsional moments:

$$M(x, y) = \begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \sum_1^n \int_{h_{k-1}}^{h_k} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}^k z dz \quad (8)$$

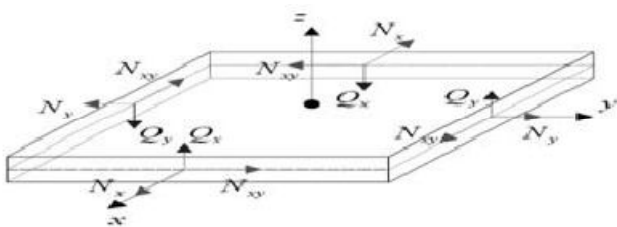


Fig. 1 Model of the plate

### D. Expressions for the Stress-Strain Plate

The stress-strain relationship is given as:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}_R^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_R^k \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix}_R^k + z \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_R^k \begin{pmatrix} \chi_{xx} \\ \chi_{yy} \\ \chi_{xy} \end{pmatrix}_R^k \quad (9)$$

and

$$\begin{pmatrix} \sigma_{yz} \\ \sigma_{xz} \end{pmatrix}_R^k = \begin{bmatrix} C_{44} & C_{45} \\ C_{45} & C_{55} \end{bmatrix}_R^k \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix}_R^k \quad (10)$$

### III. NUMERICAL SIMULATION OF THE PLATE

This part concerned a numerical simulation with abaqus software to obtain a different property of plate with crack growth initiation and give us an idea for the distribution of the stress, also we will see the strain of the model in using the FEM methods. When we create the model we have to enter the material (ALUMINIUM) properties which are:

- Young's modulus: 70 GPa
- Poison's ratio: 0.33
- Density: 2.7 Kg/m<sup>3</sup>

All numerical data concerning the mesh structure is:

The finite element model of the plate is obtained by the ABAQUS software.

TABLE I  
NUMBER OF ELEMENTS AND NODES

Number of elements	Number of nodes
2970	30213

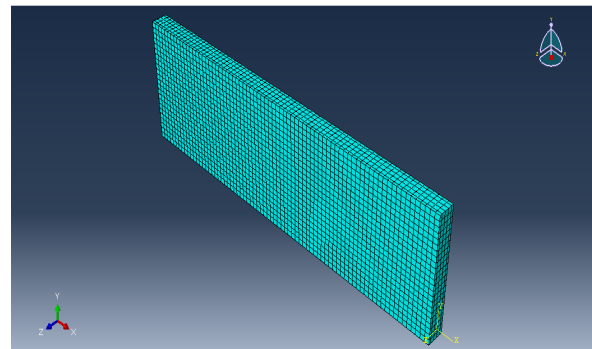


Fig. 2 Structure of the plate

### A. Frequency Study

In this section simulation can identify and indicate a set of modal deformation of the plate to determine the eigenmodes of the plate.

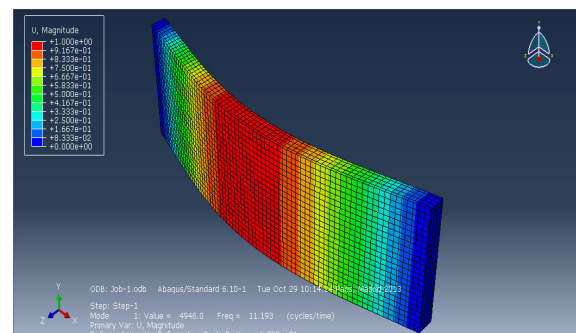


Fig. 3 The first Mode shape, f = 11.193 Hz, Umax = 1.000 mm

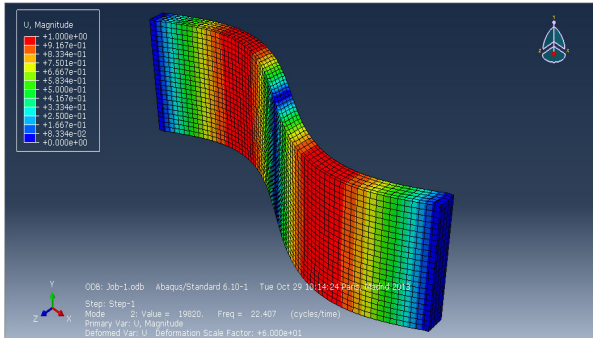


Fig. 4 The second Mode shape,  $f = 22.407$  Hz,  $U_{max} = 1.000$  mm

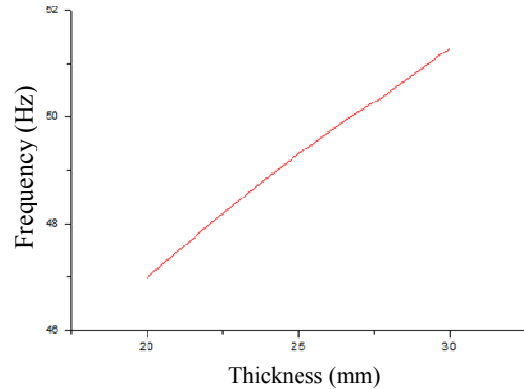


Fig. 8 Frequency function in thickness

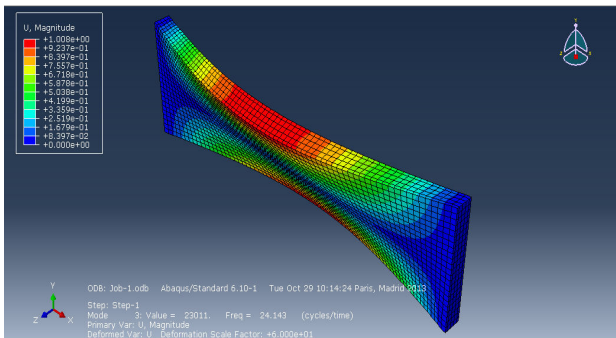


Fig. 5 The third Mode shape,  $f = 24.143$  Hz,  $U_{max} = 1.008$  mm

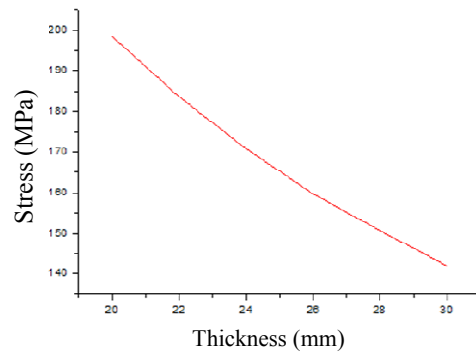


Fig. 9 Von Mises stress function in thickness

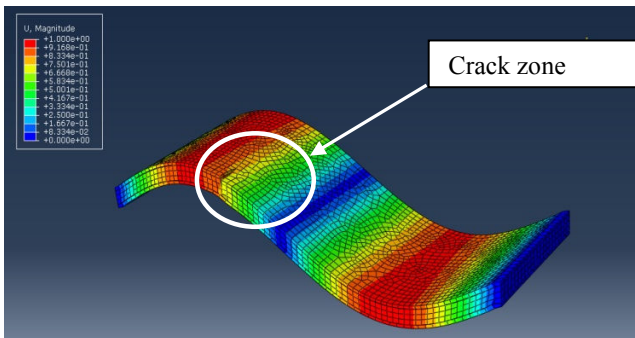


Fig. 6 The second mode shape with damage

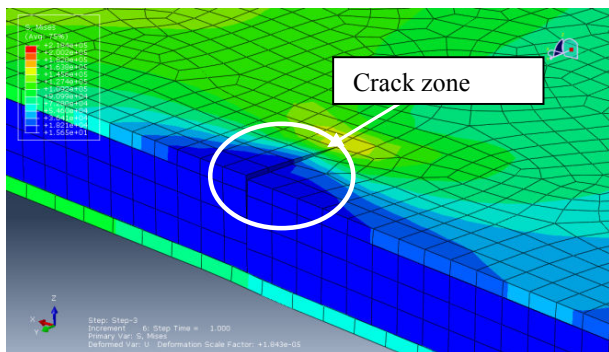


Fig. 7 Von Mises stress function damage growth

### B. The Frequency and the Von Mises Stress According the Thickness of the Plate

The graph below shows the frequencies and Von Mises stress according the thickness function of the plate.

*C. Analysis and Discussion of Results*  
 The visualization of these figures and also the simulation results (displacements, frequencies) we note later, the displacements are proportional to the frequency that involves deformations that spread when the frequency increases and this again depends on what kind strain (compression, tension, bending) it is subjected to the plate.

### IV. CONCLUSION

The topic discussed in this article is the numerical modeling of aluminum structures used for passive damping and vibration control. The frequency dependence of the nonlinear behavior introduces complexity in modeling these structures for direct and accurate determination of the frequency properties and for predicting responses. In these nonlinearities come coupling material nonlinearities geometric model in the case of these structures in large displacements.

The finite element modeling is used to simulate the static and dynamic structures whose mechanical properties of the plates are determined behavior.

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